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Calculation of stresses in filament-wound dome/cylinder joint using influence coefficients

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Presented text deals with the calculation of stresses in filament wound dome/cylinder joint where membrane stress condition is violated (cylindrical part of the vessel has different thickness than dome part). Due to this fact statically indeterminate shear force and bending moment occur in the joint (see Fig. 1 for details). These internal force effects cause inner plane resultant forces and moments acting on cylindrical part and dome of the vessel which are used for calculation of additional stresses (membrane and bending) in the joint.

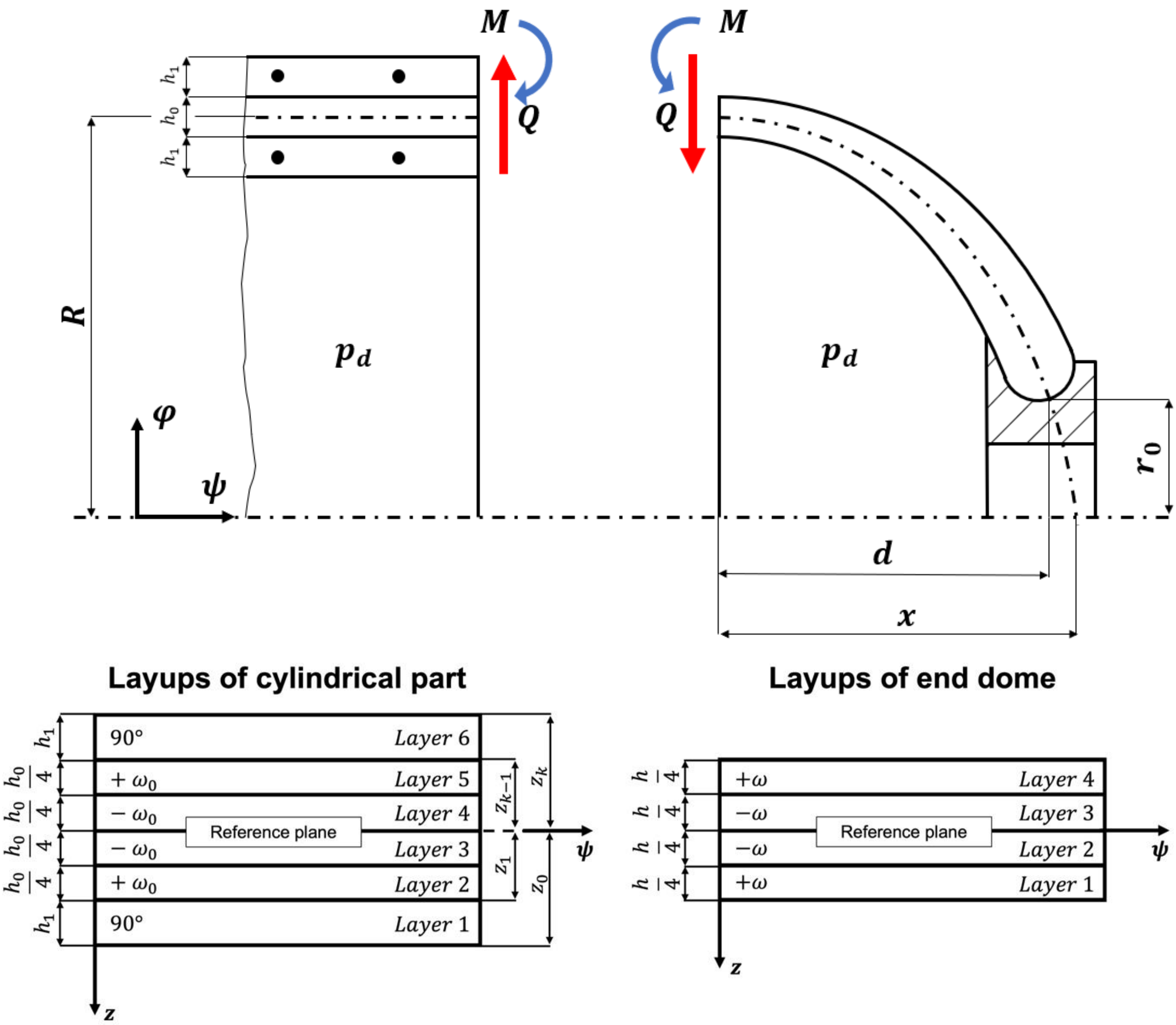

Fig. 1. Scheme of the dome/cylinder joint with internal force and composite lay-up for each part

Shear force Q and bending moment M can be calculated using influence coefficients derived in [2] as

$$M = p_d \; \frac{\left(\delta_p^C - \delta_p^D\right)\left(\vartheta_Q^D - \vartheta_Q^C\right)}{\left(\Delta_Q^C + \Delta_Q^D\right)\left(\vartheta_M^C + \vartheta_M^D\right) + \left(\Delta_M^C - \Delta_M^D\right)\left(\vartheta_Q^D - \vartheta_Q^C\right)} \;, \tag{1}$$

$$Q = M \; \frac{\vartheta_M^C + \vartheta_M^D}{\vartheta_Q^D - \vartheta_Q^C} \,, \tag{2}$$

where p_d is damage pressure and $\delta_p^{C,D}$, $\Delta_{Q,M}^{C,D}$ and $\vartheta_{Q,M}^{C,D}$ are influence coefficients [1]. Subscript indicates what is the influence factor caused by (p indicates internal pressure, Q indicates shear force and M indicates bending moment). Superscript indicates the area where the influence factor act (C indicates cylindrical part and D indicates end dome).

Inner plane resultant forces and moments are derived with the use of bending theory of cylindrical shell. Geometric equations of cylindrical shell (principal radii of curvature, normal angle to the shell midplane, etc.), equilibrium equations of the elements, equations which combine strain with displacements and curvature changes with displacements and slopes and classic lamination theory (ABD matrices and inner plane resultant forces and moments) were used for the derivation of fourth order ordinary differential equation for deflection line of the shell midplane. Boundary conditions given by shear force Q and bending moment M are used for the solution of integration constant which are used for computation of inner plane resultant forces and moments.

Strains on midplane for cylindrical part and end dome can be computed as

$$\begin{bmatrix}\varepsilon_\psi^0\\ \varepsilon_\varphi^0\end{bmatrix} = \boldsymbol{a}\left(\begin{bmatrix}N_\psi\\ N_\varphi\end{bmatrix}_p + \begin{bmatrix}N_\psi\\ N_\varphi\end{bmatrix}_Q + \begin{bmatrix}N_\psi\\ N_\varphi\end{bmatrix}_M\right), \tag{3}$$

where $\boldsymbol{\varepsilon_0}$ are strains on midplane, $\boldsymbol{N_p}$ are in-plane resultant forces caused by membrane stresses, $\boldsymbol{N_Q}$ are in-plane resultant forces caused by shear force Q, $\boldsymbol{N_M}$ are in-plane resultant forces caused by bending moment M and $\boldsymbol{a}$ is the inverted tensile stiffness matrix (for cylindrical part is $\boldsymbol{a} = \left(A_{ij}^C\right)^{-1}$and for end dome is $\boldsymbol{a} = \left(A_{ij}^D\right)^{-1}$).

The curvature changes of the midplane can be computed as

$$\begin{bmatrix}\kappa_\psi\\ \kappa_\varphi\end{bmatrix} = \boldsymbol{d}\;\begin{bmatrix}M_\psi\\ M_\varphi\end{bmatrix}_M \;, \tag{4}$$

where $\boldsymbol{\kappa}$ are curvature changes of the midplane, $\boldsymbol{M_M}$ are in-plane resultant moments caused by bending moment M and $\boldsymbol{d}$ is the inverted bending stiffness matrix (for cylindrical part is $\boldsymbol{d} = \left(D_{ij}^C\right)^{-1}$and for end dome is $\boldsymbol{d} = \left(D_{ij}^D\right)^{-1}$).

Total strain in meridian and circumferential directions can be computed as

$$\begin{bmatrix}\varepsilon_\psi\\ \varepsilon_\varphi\end{bmatrix} = \begin{bmatrix}\varepsilon_\psi^0\\ \varepsilon_\varphi^0\end{bmatrix} + z_k\;\begin{bmatrix}\kappa_\psi\\ \kappa_\varphi\end{bmatrix}, \tag{5}$$

where z_k is the distance from the reference plane to the surface of the *k*-th layer (see Fig. 1). Then the stresses in the *k*-th layer in meridian and circumferential directions can be written as

$$\begin{bmatrix}\sigma_\psi\\ \sigma_\varphi\\ \tau_{\psi\varphi}\end{bmatrix}^k = \boldsymbol{Q}^k\begin{bmatrix}\varepsilon_\psi\\ \varepsilon_\varphi\\ 0\end{bmatrix}^k \;, \tag{6}$$

where σ_ψ and σ_φ are normal stresses, $\tau_{\psi\varphi}$ is shear stress and $\boldsymbol{Q}$ is the well-known reduced stiffness matrix (for cylindrical part is $\boldsymbol{Q}^k$=Q_{ij}^k and for end dome is $\boldsymbol{Q}^k$=$\overline{Q_{ij}}$).

These resulting stresses ($\sigma_\psi, \sigma_\varphi$ and $\tau_{\psi\varphi}$) in coordinate system (ψ, φ) are created by superposition of the membrane stress, the stress caused by shear force Q, the stress caused by bending moment M and the stress caused by change in curvature as shown in Fig. 2 (cylindrical part) and Fig. 3 (end dome). There are six points on the cylindrical part (A_C, B_C, C_C, D_C, E_C and F_C) and two points on end dome (C_D and D_D) where peak stresses can occur (see Fig. 2 and Fig. 3) [3].

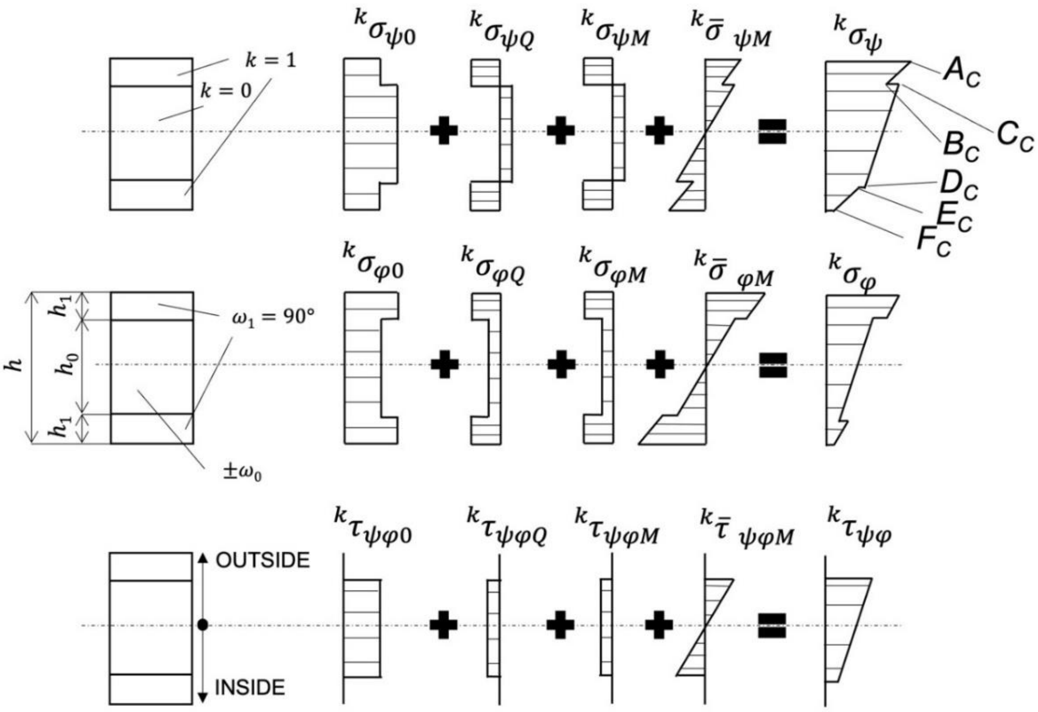

Fig. 2. Superposition of stresses for cylindrical part

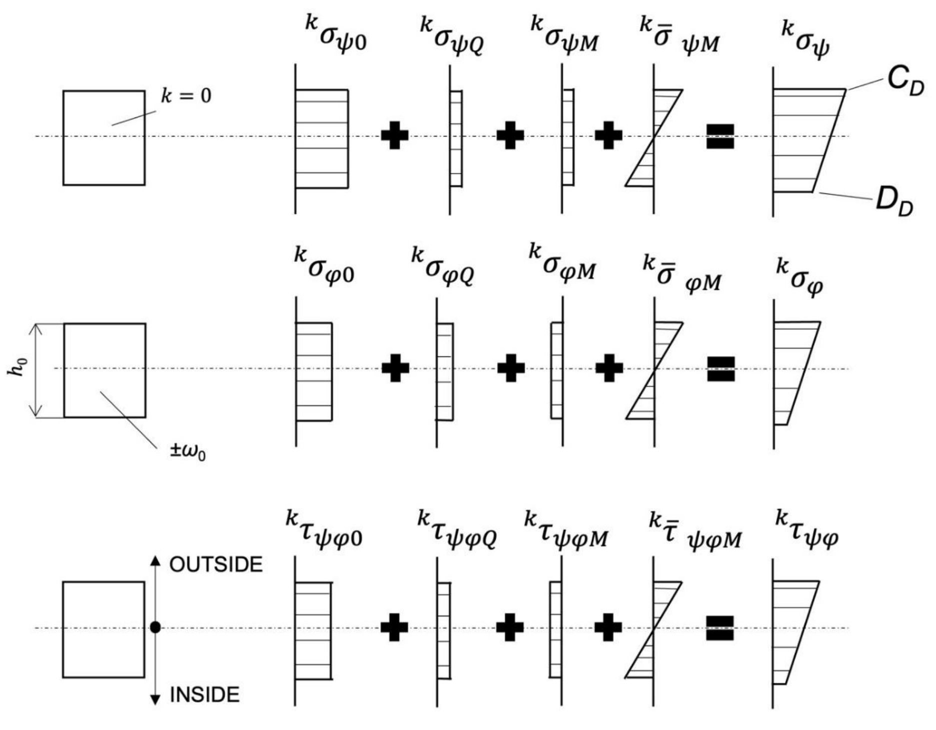

Fig. 3. Superposition of stresses for dome part

Stresses in coordination system (ψ, φ) are transformed to material coordinate system (L, T). This transformation can be written as

$$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2\,m\,n \\ n^2 & m^2 & -2\,m\,n \\ -m\,n & m\,n & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_\psi \\ \sigma_\varphi \\ \tau_{\psi\varphi} \end{bmatrix}, \tag{7}$$

where $m = \cos\omega$ and $n = \sin\omega$. The same superposition can be used for resulting stresses (σ_L, σ_T and τ_{LT}) in material coordinate system (L, T). In each point shown in Fig. 2 and Fig.3 failure of the pressure vessel can occur. Therefore, all these stresses were used for failure index (FI) computation (see Fig. 4) for given internal pressure 1 MPa according to Hoffman strength criterion (see [1] for example). In Fig. 4 is presented FI for the cylinder joint with geodesic-isotensoid dome.

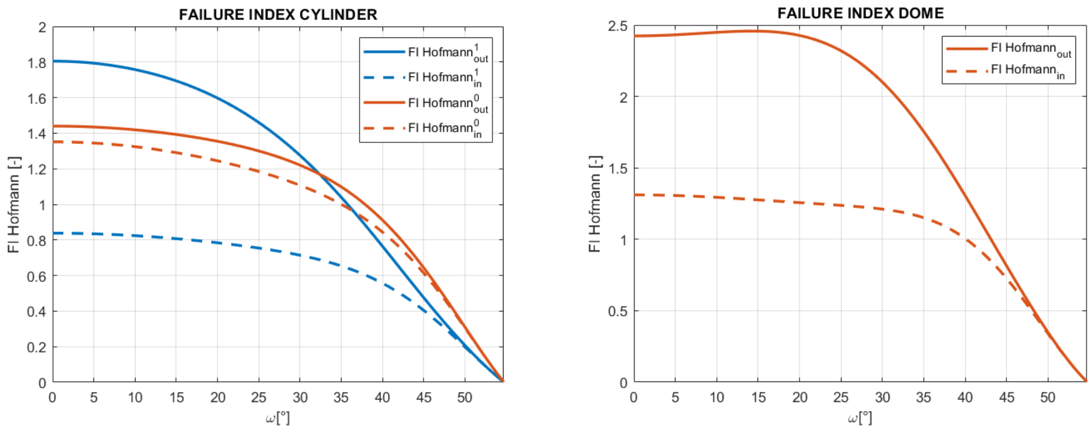

Fig. 4. FI for the cylinder joint with geodesic-isotensoid dome

This study describes a procedure for fast and efficient analysis of the junction between the cylindrical part and end dome of a filament wound pressure vessel. Analysis was done for five types of domes (sphere, geodesic-isotensoid, dome with zero transversal stress, dome with zero transversal strain and dome with identical strains). From the results we may see which dome is the best for the connection from the failure index point of view. Future work will focus on multi-criteria optimization of the dome shape according to the joint with cylinder.

Acknowledgements

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