Determination of wall heat transfer coefficient of a cylindrical annulus

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1 Introduction and formulation of the problem

The wall heat transfer coefficient is one of the key parameters for the numerical solution of temperature deformation problems. In this contribution, a semi-analytical method is presented for determining the wall heat transfer coefficient of a cylindrical annulus assuming a fully developed steady laminar velocity profile corresponding to Poiseuille flow $u(r)$ and a steady developing fluid temperature profile.

Investigated cylindrical annulus of length L formed by an inner cylinder of radius r_i and an outer cylinder of radius r_o is in Fig. 1. Boundary conditions of constant inlet temperature T_e of the fluid and constant temperature of the inner wall T_i and the outer wall T_o of the cylindrical annulus are considered.

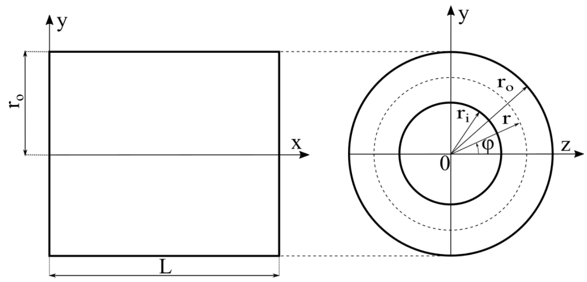

Figure 1: Geometry of the considered computational area.

2 Determination of temperature and wall heat transfer coefficient

The starting point of our investigation is the energy equation in the form of

$$u(r)\frac{\partial T}{\partial x} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right), \tag{1}$$

where α is thermal diffusivity of the fluid. Using a method inspired by Viskanta (1964), the energy equation was converted to the problem of solving equation of the Sturm-Liouville type. The eigenvalues λ, the eigenfunctions $R(\xi)$ and the derivatives of the eigenfunctions $\frac{\mathrm{d}R(\xi)}{\mathrm{d}\xi}$ of the Sturm-Liouville equation were solved numerically by the method of finite volumes, see Siedlecki at al. (2016). We present the solved temperature function in the form

$$T = \left[\sum_{n=0}^{\infty} C_n R_n(\xi) e^{-\lambda_n^2\zeta} + 1 - \frac{\ln\xi}{\ln\xi_i}\right](T_e - T_i) + T_i + T_o - T_e + \left[\sum_{n=0}^{\infty} D_n R_n(\xi) e^{-\lambda_n^2\zeta} + \frac{\ln\xi}{\ln\xi_i}\right](T_e - T_o), \tag{2}$$

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where $\xi = \frac{r}{r_o}$ and $\zeta = \frac{(1-\xi_i)x}{Pe(r_o-r_i)}$ are dimensionless coordinates, C_n and D_n are constants determined from the boundary conditions by Moore–Penrose inverse and $\xi_i = \frac{r_i}{r_o}$.

The wall heat transfer coefficient of the inner wall h_i, or the wall heat transfer coefficient of the outer wall h_o, was calculated by substituting into Newton's law of cooling.

3 Selected results and conclusion

The semi-analytically derived temperature and wall heat transfer coefficients are compared with the numerical results provided by the professional computing system Ansys CFX when solving the same problem specified by concrete values of geometric parameters and boundary conditions. Definition of the solving example was provided by Doosan Škoda Power s.r.o.

It can be stated that the values of the wall heat transfer coefficient determined using the methodology presented here are in a very good agreement with the values that were determined numerically using the professional computing system Ansys CFX. Fig. 2 shows the wall heat transfer coefficient of the inner and the outer wall.

Investigating the flow of an incompressible liquid in a cylindrical annulus is the first step in the complex work of establishing the methodology for determining the wall heat transfer coefficient. Follow-up steps will be the investigation of the wall heat transfer coefficient in more complex geometries during transitional or turbulent steam flow regimes in cooperation with Doosan Škoda Power s. r. o.

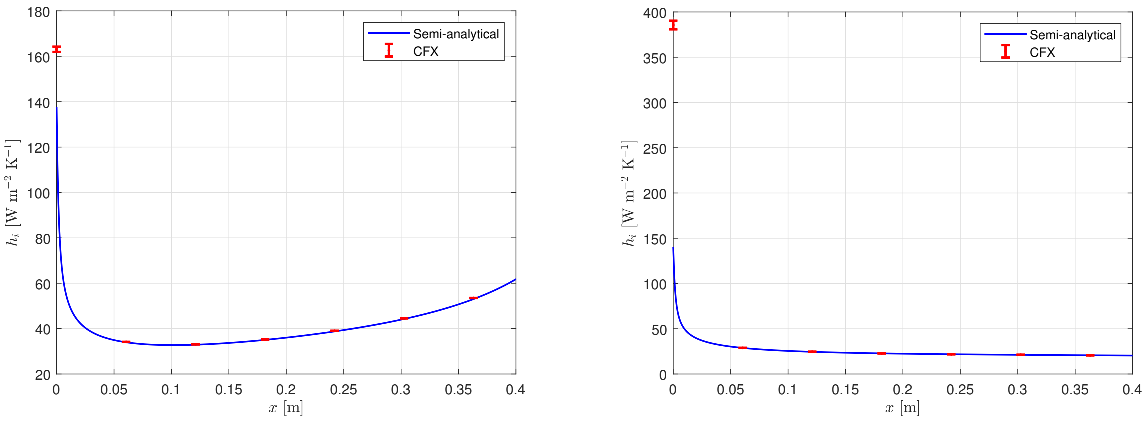

Figure 2: Heat transfer coefficient through the inner wall h_i (left) and the outer wall h_o (right) of the cylindrical annulus.

Acknowledgement

This work was supported by the student grant project SGS-2022-008.

References

Viskanta, R. (1964) Heat transfer with laminar flow in a concentric annulus with prescribed wall temperatures *Applied Scientific Research, Section A*. Argonne National Laboratory, Illinois.

Siedlecki, J. and Ciesielski, M., Blaszczyk, T. (2016) The Sturm-Liouville eigenvalue problem: A numerical solution using the Control Volume Method. *Journal of Applied Mathematics and Computational Mechanics*, Volume 2016, pp. 127–136.