FACULTY OF APPLIED SCIENCES
UNIVERSITY
OF WEST BOHEMIA

DEPARTMENT OF
MATHEMATICS

Master's Thesis

Cost Optimisation of LED Lighting in Vertical Farms

Nikola Svrčinová

PILSEN, CZECH REPUBLIC

2024

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(projektu, uměleckého díla, uměleckého výkonu)

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Zásady pro vypracování

1. Seznamte se s pokročilými optimalizačními metodami pro vícekriteriální optimalizaci.
2. Proveďte rešerši zaměřenou na metodiku využívání umělého LED osvětlení pro pěstování vybraných druhů plodin.
3. Navrhněte optimalizační modely pro úlohu minimalizace nákladů.
4. Implementujte modely ve vhodném prostředí a srovnejte různé metodiky pěstování u vybraných druhů plodin.

Rozsah diplomové práce: **40-80 stran**
Rozsah grafických prací: **dle potřeby**
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Seznam doporučené literatury:

- EHRGOTT, Matthias. Multicriteria Optimization. Heilderberg: Springer, 2005. isbn 3-540-21398-8.
- HIROKI, R. et al. Identifying the Optimum Light Cycle for Lettuce Growth in a Plant Factory. Acta Hortic. Available also from: https://doi.org/10.17660/ActaHortic.2014.1037.115.
- XYDIS, George; AVGOUSTAKI, Dafni Despoina. Energy cost reduction by shifting electricity demand in indoor vertical farms with artificial lighting. Biosystems Engineering. Available also from: https://doi.org/10.1016/j.biosystemseng.2021.09.006.
- Další literatura bude upřesňována průběžně.

Vedoucí diplomové práce: **Doc. Ing. Roman Čada, Ph.D.**
Katedra matematiky

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Západočeská univerzita v Plzni
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V Plzni dne 2. října 2023

Declaration

I hereby declare that this Master's Thesis is completely my own work and that I used only the cited sources, literature, and other resources. This thesis has not been used to obtain another or the same academic degree.

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In Pilsen, on 20 May 2024

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Nikola Svrčinová

Abstract

This Master's thesis is focused on optimising the costs of lighting in indoor vertical farms. First, it explains the condition for crop growth and the operation of LED lights in vertical farming. Second, it introduces possible models of electricity savings for consumers on day-ahead energy markets. The thesis then describes a multi-criteria problem involving different lighting strategies and light sources. A set of Pareto-optimal lights is found, and one lamp is selected by the weighted sum method. The programming part is solved in Matlab. Electrical energy saving models are programmed in addition in Industrial Edge applications by the company Siemens.

Abstrakt

Diplomová práce se zaměřuje na minimalizaci nákladů na osvětlení ve vertikálních farmách. Nejprve jsou vysvětleny podmínky pro pěstování rostlin a fungování LED světel v oblasti vertikálního farmaření. Dále jsou představeny možné modely pro šetření elektrické energie pro odběratele na day-ahead energetických trzích. Pro kombinace různých modelů svícení a lamp je popsána mutikriteriální úloha. V úloze je nalezena množina pareto-optimálních lamp a pro výběr jedné lampy je použita metoda vážených sum. Programovací část práce je řešena v prostředí Matlab. Modely šetření elektrické energie jsou navíc naprogramovány v aplikacích platformy Industrial Edge od firmy Siemens.

Keywords

indoor vertical farm • day-ahead energy prices • LED lighting • multi-objective optimisation • Pareto-optimal set • weighted sum method

Acknowledgement

I would like to thank doc. Ing. Roman Čada, Ph.D., for all his useful advice and consultations regarding the mathematical point of view of this thesis and the form of the work. I would like to express my great thanks to Ing. Tomáš Svoboda for his creative ideas and dedicated time during consultations. Finally, I would like to thank my family and friends for their support and encouragement throughout my studies.

Contents

Glossary and Notation

$\mathbf{c}$	Vector of day-ahead prices
CC	Constant continuous strategy
CI	Constant intermittent strategy
DC	Dynamic continuous strategy
DI	Dynamic intermittent strategy
DLI	Daily light integral
$f_i(x)$	i-th objective function
HLI	Hourly light integral
χ	Feasible set
χ_E	Set of all nondominated points
IVF	Indoor vertical farm
LED	Light emitting diode
LP	Linear programming
$\mathbb{N}$	Natural numbers
PAR	Photosynthetically active radiation
PPF	Photosynthetic photon flux
PPFD	Photosynthetic photon flux density
$\mathbb{R}$	Real numbers
SAW	Simple additive weighting sum method
w_i	Weight of i-th objective function
$\hat{x}$	Efficient or Pareto-optimal solution
z	Paid price for one day

Introduction 1

1.1 What is vertical farming?

Indoor vertical farming (IVF) is a way of growing plants in a fully isolated environment, which replaces sunlight with artificial lighting. As it can be seen in Figure 1.1, plants are grown by stacking layers, thus saving space for one unit area. IVFs are placed near urban areas, where land is limited or expensive. Growing crop near cities allows reducing transportation costs and carbon footprint. To precisely regulate the environment in IVF, climate controllers are used to regulate temperature, humidity, CO_2, nutrition, speed of air and other influences on growth based on the type of plant. The controlled environment ensures the whole year production of consistent and high-quality products [1].

Figure 1.1: AeroFarm's vertical farm [2].

There is other type of indoor farms, which are called greenhouses. Greenhouses have artificial lighting together with natural solar radiation since the walls are permeable. The plants could be grown there in one layer, so the sunlight is spread evenly.

This thesis focuses on optimising lighting costs only of indoor vertical farms since in greenhouses, the attention has to be paid to different factors as weather forecasts.

1.2 Motivation

With the increasing population also increases the demand for food production. Compared to field agriculture, IVFs reduce excessive use of pesticides, soil erosion and keep the surrounding ecosystem in balance. IVFs could be crucial in places with a long period of darkness or cold [3]. On the other hand, IVFs have negative environmental impacts without using renewable energy as an energy source. Therefore, the thesis is focusing on purchasing electricity from the day-ahead energy markets since it depends on renewable energy.

Vertical farm experts created a design of vertical farm for leafy green or vine crop production [4]. The annual costs of this vertical farm are shown in pie chart 1.2. 42.6 % of the chart constitute by energy costs. Energy consumption consists of illumination, air management, nutrient delivery, health monitoring and other systems. The Figure 1.3 shows decomposition to individual systems of energy costs. Illumination system creates almost 30 % of all expenses and that is the main reason why this thesis is focusing on minimising the costs of lighting.

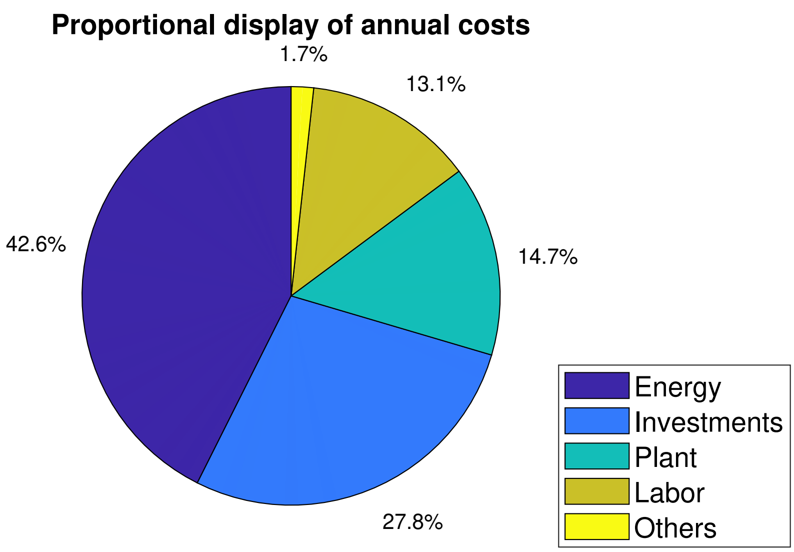

Figure 1.2: Pie chart of yearly costs of the vertical farm with leafy greens and vine crop [4].

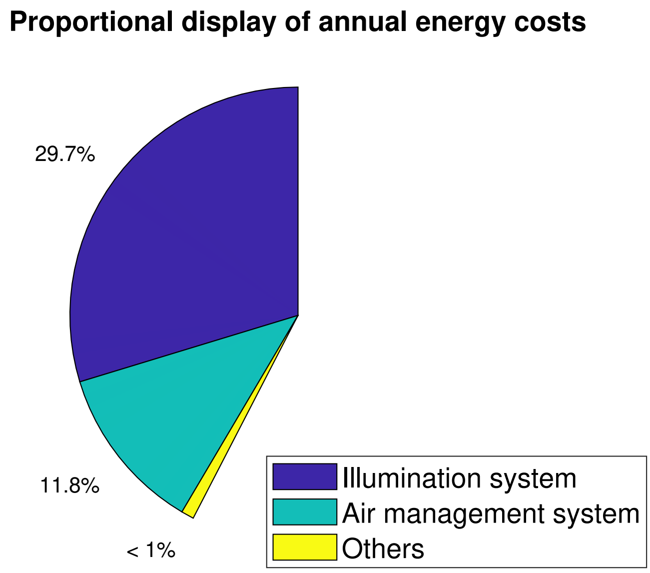

Figure 1.3: Pie chart of yearly energy costs of the vertical farm with leafy greens and vine crop [4].

The controlled environment is demanding for technology. Specialised equipment and controllers have high initial cost, maintenance and managing all the equipment needs the necessary knowledge of experts. This thesis will focus only on optimising the costs of lights, based on the lighting strategy and light parameters such as price of the light, number of lights, colour possibilities of the light bulb and light degradation.

1.3 Work overview

The thesis starts with explaining crop conditions for growth. First, light intensity conditions for lettuce in different stages of growth are shown, as well as comparison of intensity conditions of some wired crop, potted and cut flowers. The thesis also takes into account colour light conditions. The thesis should answer the question as follows

- What is the required intensity and colour for specific crops?

Secondly, types of LED lights that are used in IVFs are introduced. The work is then dedicated to top linear lights, which is one type of LED lights, and it can be used in all vertical farms. Next is estimated the number of lights that are needed in IVF that does not already have a fixed number of light customised by light producer. Then the presented thesis focuses on calculation of energy consumption and light degradation. The degradation is estimated based on LED lifespan. The thesis focuses on questions as follows

- What is the smallest amount of LED lights to minimise initial costs based on the distance between light and canopy?
- Is it financially more advantageous to have fewer lights with high intensity or more lights with low intensity?
- Which LED lights need to be replaced more frequently due to their quicker light degradation?

The Chapter 3 starts with explaining energy contracts, specifically the ones with day-ahead energy prices. Afterwards, parameters, which are used in lighting strategies to save costs of energy, are introduced. These strategies differ in intensity, total lighting time, light cycles and day length. The main focus is on describing 2 different strategies of light intensity. The first is constant, and the second is called dynamic strategy. Constant lighting has the same intensity during a day, meanwhile, the dynamic strategy has different intensity each hour. Algorithms and solution are described and can be found in an enclosed Matlab script `model_code.m`. The models are compared with a model, where is no optimisation is done. Implementation of lighting strategies is briefly introduced in Siemens applications LiveTwin, Flow Creator and Energy Manager. Apart from Matlab, which is used only for research reasons, above mentioned applications are used in practice for predictive system control and optimisation.

The last Chapter 4 covers multicriterial optimisation, which should answer the following question:

- What is the optimal LED light and lighting strategy for a chosen crop based on the crop conditions and specific light parameters?

Input parameters of strawberries and romaine lettuce are chosen for comparing results of multicriterial problem. These two are chosen because of their different intensity and required number of hours of light. First, a set of optimal lights is found based on Pareto-optimality. The computation is in file `bruteforce.m`. Second, different normalisation techniques and weights are used in weighted sum method. The programming part is in attached file `multicriterial.m`. Final, the results are described.

1.4 Resources

All written resources of this thesis are listed in the thesis. There was an opportunity to speak with representatives of vertical farms, a lighting company, and a specialist in the energy market. The first information and introduction to the topic was inspired by talks with representatives of vertical farms from Nordic Harvest and GreeenTech. The estimated number of light was inspired by information from a representative of lighting company Food Autonomy.

Methodology of growing crop under LED lighting

2

The primary goal of this chapter is to show the parameters, which are relevant to growing crops under LED lighting. These parameters will be used in Chapter 4 for multicriterial optimisation. Firstly, parameters connected with crop conditions such as intensity and colour are introduced. Next, types of LED lights are shown. From these types, the linear top light is chosen, since it can be used in any vertical farm. The number of lights in IVF and energy consumption for one day are estimated for top lights. The last part of this chapter is devoted to approximating light degradation for the constant and dynamic lighting strategies, which are described in Chapter 3.

2.1 Crop conditions

2.1.1 Intensity of LED light

Growing conditions are different from crop to crop and stage of their development, see an example in Table 2.1. In terms of light conditions during growing, enough light intensity and appropriate colour spectrum needs to be ensured. The light intensity is measured in moles per specific area and time using the Daily Light Integral (DLI), which is the total amount of moles per day on 1 meter squared. Formally DLI $\in \mathbb{R}^+$ in $\frac{\text{mol}}{\text{m}^2\text{d}}$. Values of DLI range mostly under 50 $\frac{\text{mol}}{\text{m}^2\text{d}}$ and very common values are between 6-18 $\frac{\text{mol}}{\text{m}^2\text{d}}$ [5]. DLI is a cumulative quantity, and therefore does not measure the instantaneous intensity. A similar quantity describing the amount of light is Photosynthetic Photon Flux Density (PPFD). PPFD $\in \mathbb{R}^+$ and it is measured in $\frac{\mu\text{mol}}{\text{m}^2\text{s}}$. PPFD describes the number of moles that fall on 1 meter squared per second. The relationship between DLI and PPFD is described as

$$\text{DLI} = t_p \text{PPFD} \frac{3600}{10^6}, \tag{2.1}$$

where photoperiod $t_p \in \langle 0, 24\rangle$ is a number of hours that the light source is on during one day.

It can be seen in Table 2.2 that wired crops require higher values of PPFD than potted crops. The cause of which is that the generative stage requires higher light intensity. The crop is not only sensitive to intensity, but also to colour and time of light. The crop can be divided into 'long day plants' and 'short day plants'. Soybean, tobacco, chrysanthemum, or cannabis sativa, as 'short day plants', need only about 8–12 hours of light and continuous dark for 14–16 hours. Tomatoes, pea plants, rose, cannabis ruderalis do not require specific day length. Spinach, radish, hibiscus, wheat, and lettuce, as 'long day plants', require about 14–18 hours of light period [6].

Table 2.1: Light condition during lettuce life cycle [4].

Stage	Days	PPFD	DLI (16 h)	intensity [%]
Germination Phase 1	1,5-2	150	8,64	60
Germination Phase 2	14	200	11,52	80
Grow Phase 1	10	200	11,52	80
Grow Phase 2	9	225	12,96	90
Grow Phase 3	9	250	14,4	100

Table 2.2: Light condition for different types of crops [5].

wired	PPFD	potted	PPFD	cut	PPFD
Pepper	230	Rose	50	Rose	220
Cucumber	230	Bromelia	90	Tulip	60
Tomato	270	Kalanchoë	90	Gerbera	90
Cannabis Vegetative	350	Orchid	160	Freesia	90
Cannabis Flowering	1 000	Dendrobium	230	Lily	90

2.1.2 Colour of LED light

The quality and appearance of a crop is influenced by the colour of light. Crops contain pigments such as chlorophylls, carotenoids, anthocyanins, phytochromes, etc. These pigments serve an essential function in photosynthesis, protecting leaves and more. They absorb light at various wavelengths and gives crops their unique colours. For example, chlorophyll does not absorb the wavelength range of green. Instead, the wavelength is reflected, and that gives plants such as lettuce their green colour [7]. In the same way, carotenoids give plants orange colour, anthocyanins purple, etc. [8].

The absorption of pigment depends on specific wavelengths. Figure 2.1 shows the general absorption of pigments based on wavelength. The y-axis shows the ratio of absorbed photons on the number of photons emitted by specific wavelength. The peaks represent the wavelength at which crops absorb light the most efficiently.

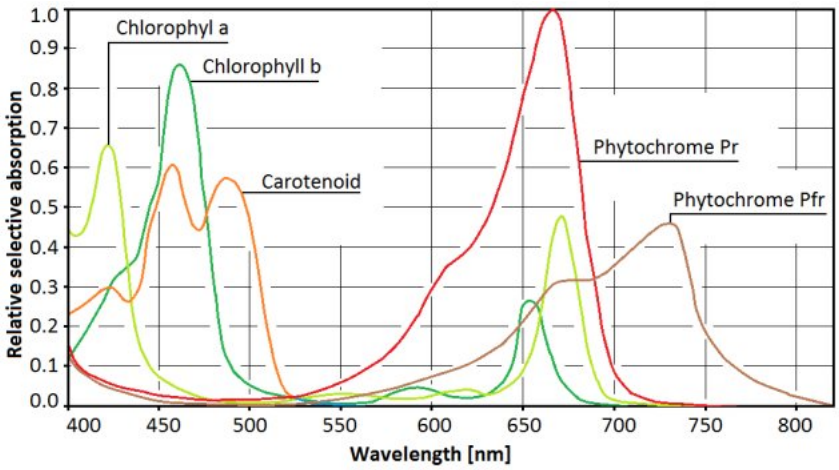

Figure 2.1: Wavelength absorption of chlorophyll a, b and beta-carotene [9].

Generally, the curves had two peaks around 440 and 650 nm for all species and conditions [10]. This wavelength represents blue and red colours. These two colours play a major role for chlorophylls, which are essential for crop growth and photosynthesis. Other wavelengths also have a complex impact on plant physiology and morphology [11]. There are general properties of colours and their influence on crops:

- ultraviolet

  Ultraviolet (UV) light has both positive and also negative effects. It increases production of flavonoids and anthocyanins, which increase crop defence mechanism against herbivore attacks and pathogens. However, these types of mechanisms are not needed in IVFs. It also negatively affects the growth, development, and yield of plants [12].

- blue

  Blue light increases the production of chlorophyll and plays key role in photosynthesis. Some growers expose leafy greens (lettuce, spinach, kale, ...) to blue light before harvest to intensify their green colour [13].

- green

  Green light seems to have fewer benefits than other colours, since it is mostly not absorbed and is instead reflected [8]. However, research shows that a small amount of this light is beneficial since it penetrates deep into plants, and it drives photosynthesis where other colours cannot.

- orange

  Orange light increases the production of carotenoids [8]. These pigments give plants like tomatoes their red, yellow and orange colours. It also promotes flowering and fruiting.

- red and far-red (FR)

  Phytochrome absorbs red and FR lights. These colours stimulate seed germination, root development, tuber or bulb formation, and it is essential for flowering and fruit formation [8].

Plants under monochromatic lights, such as only blue or only red LEDs, reduce photosynthetic rate and have growth abnormalities [11]. A proper energy balance of red and blue spectral regions helps plants to have normal growth and form. Lettuce under white+far-red light showed increase in fresh and dry mass in comparison with white and white+blue light. Although it also showed decreases in some antioxidants (vitamin C, flavonoids, phenolic compounds). [14].

Except LED lights with colour channels, white light are often used. White LEDs provide the full spectrum of light and are easy to use. They are usually sufficient for growing lettuce at home or on small scale. Lights with specific colours allow adjusting the intensity of each colour and are more efficient for large operations.

2.2 Types of LED lights

LED lights have a lifespan range of 10,000 to 50,000 hours, which is higher in comparison to other lights [15]. They emit little to no heat, and their light can be directed perfectly. In comparison to other types of light they have smaller energy costs, and they do not burn out so easily, instead, they experience lumen depreciation [16].

Before going to production, there exist top research lights like Attis 300W from the company Food Autonomy, pictured in Figure 2.2 [17]. Research lights offer a wide colour and intensity range, however, they have a very high power consumption and are used only in laboratories. For leafy vegetables, linear top lightings are used, see Figure 2.3. While for wire vegetables and soft fruits, interlightings are also used, shown in 2.4 [18]. Interlighs are hanged between rows of crop to illuminate lower and shadowed parts of crops. To grow cut flowers, potted plants or perennials, top lights and special flowering light bulbs are used.

Figure 2.2: Food Autonomy Horticultural lighting for research.

Figure 2.3: Philips linear top lighting.

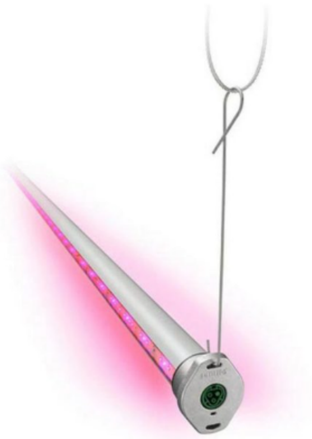

Figure 2.4: Philips interlight for wire vegetables and soft fruit.

2.3 Top lighting

2.3.1 PPF

To calculate initial expenses for IVFs, the number of lights needed in one layer of the factory area has to be determined. Besides photosynthetic photon flux density

(PPFD), there also exists photosynthetic photon flux (PPF) $\in \mathbb{R}^+$ in $\frac{\mu\text{mol}}{\text{s}}$. As stated before, PPFD is the number of moles falling on a surface of 1 meter squared per 1 second. On the other hand, the PPF describes the total amount of moles that are emitted by a light source each second in the photosynthetically active radiation (further PAR) zone. PAR is part of the electromagnetic radiation with wavelengths between 400-700 nm, which is utilised by plants for photosynthesis. The PPF is measured by an integrating sphere, and it is stated by lighting producers in datasheets.

In this section, the number of lights for vertical farm for a specific area are estimated. The calculations are not precise since they do not involve information about surface reflections, and they neglect different types of light distributions. If the light is uniformly distributed in all directions, the relationship between PPFD and PPF could be described by the inverse square law. Generally, the law states that

$$I = \frac{S}{r^2},$$

where $I \in \mathbb{R}^+$ is the intensity of the flux, $S \in \mathbb{R}^+$ is the source strength and $r \in \mathbb{R}^+$ is the distance. The law states that, if a source spreads the flux S equally on a sphere, the intensity of the flux I is inversely proportional to the square of the radius of the sphere. The relationship is graphically depicted in Figure 2.5. For interlight and flowering light bulbs, which spread the light on a sphere, the intensity on surface (PPFD) is calculated as

$$\text{PPFD} = \frac{\text{PPF}}{4\pi r^2}, \tag{2.2}$$

where r is the radius of circle.

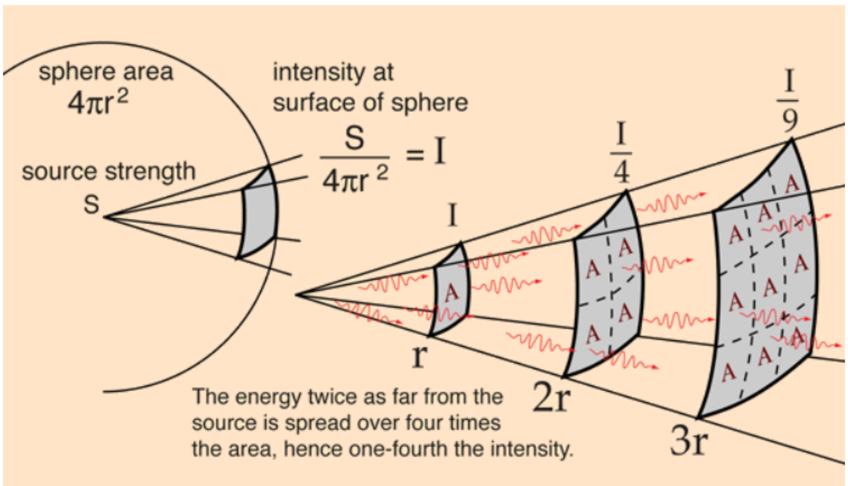

Figure 2.5: Graphical representation of Inverse Square Law [19].

LED lights have high uniformity and can be very easily directed since each chip has a typical emission angle of about 120-150 degrees [20]. In light datasheets, 120 degrees as standard beam width and 150 degrees as wide beam width are most often stated. For the calculation of the top light example, an assumption that the whole top lighting is spreading its light only to rectangle area under specified beam width is made. The second assumption is, that the lights in one line follow each other without any extra space between them.

The equation 2.2 calculated intensity for a point source which spread the light on a sphere. Top linear LED lighting spreads light on the surface of a cylinder, which changes the calculation of intensity to

$$\mathrm{PPFD}(v) = \frac{\mathrm{PPF}}{b_w l_l v}, \tag{2.3}$$

where $\mathrm{PPFD}(v)$ intensity on cylinder surface, which depends on distance v,
PPF is photosynthetic photon flux of light,
b_w is beam width in radians,
l_l is the length of light in meters,
v is the distance between light and point, where was measured the intensity PPFD.

2.3.2 Number of lights in the cultivation area

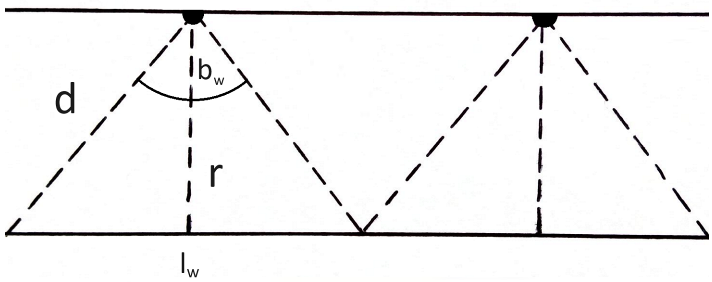

Figure 2.6: Visual description of quantities used in section 2.3.

The area where light falls has length l_l and width l_w, which is calculated as

$$l_w = 2\tan\left(\frac{b_w}{2}\right)r, \tag{2.4}$$

where r is distance between light and soil or canopy. The quantities are depicted in Figure 2.6 for quicker orientation. The calculation of number of lights assumes that each surface of a cultivation area has to have the exact $\mathrm{PPFD}_{\mathrm{crop}}$ or higher. The light intensity is calculated right under the light with distance r and on the edges of beam span with distance d. To calculate the number of light, several possibilities are considered

- $\mathrm{PPFD}(r) < \mathrm{PPFD}_{\mathrm{crop}}$

  The intensity $\mathrm{PPFD}(r)$ on the floor right under the light for fixed distance is lower than the needed $\mathrm{PPFD}_{\mathrm{crop}}$. This means that the crop is not receiving enough light intensity and the light is not appropriate. Therefore, a light with higher PPF or lowering the light closer to the canopy is needed.

- $\mathrm{PPFD}(d) > \mathrm{PPFD}_{\mathrm{crop}}$

  This means that PPF of the light is strong and the intensity of light could be lowered to $\frac{\mathrm{PPFD}_{\mathrm{crop}}}{\mathrm{PPFD}(d)}$ % of initial intensity.

- $(\mathrm{PPFD}(d) < \mathrm{PPFD}_{\mathrm{crop}}) \wedge (\mathrm{PPFD}(r) > \mathrm{PPFD}_{\mathrm{crop}})$

  The light intensity is not enough over the whole beam span. It means that the intensity is set to 100 % and the distance between lights is smaller. The distance which fulfils the required PPFD is found as $v = \frac{\mathrm{PPF}}{\mathrm{PPFD} b_w l_l}$ from 2.3. The light width is then $l_w = 2\sqrt{r^2 - d^2}$.

One light area A is a multiple of light length and width, $A = l_l l_w$. The number of lights N_l is equal to the ratio of the cultivation area C with the area A rounded up to whole numbers, $N_l = \lceil \frac{C}{A} \rceil$.

2.3.3 Relationship between the number of lights and the distance of light and floor

The relationship between the number of lights and the distance between the light and the floor is depicted in Figure 2.7 for $\mathrm{PPF} = 250\ \frac{\mu\mathrm{mol}}{\mathrm{s}}$ and $\mathrm{PPFD} = 156.25\ \frac{\mu\mathrm{mol}}{\mathrm{m}^2\mathrm{s}}$.

For small distances between 0 and 0.3 m the $\mathrm{PPFD}(d)$ is greater than the minimal PPFD ($\mathrm{PPFD}(d) > \mathrm{PPFD}_{\mathrm{crop}}$). In this case the further away the lights are from the floor the higher the bean span is and the IVF therefore needs fewer lights. From 0.3 to 0.6 m, the lights are so far away from the floor that to preserve minimal light intensity the light has to be closer to another light. It increases the number of lights. If the lights are further than 0.6 m, even if the lights are on 100 % intensity, the area under light is not sufficiently illuminated.

The price of energy for one day based on the distance between light and canopy can be seen in Figure 2.8. The price of energy for distance between 0 and 0.3 m remains constant. With shorter distance, the number of lights increases, however the light intensity is lower. That makes the price the same. The price of energy for distance between 0.3 and 0.6 m is increasing. The amount of light is increasing only slightly in comparison to shorter distance, however all the lights are now working on 100 % of their intensity.

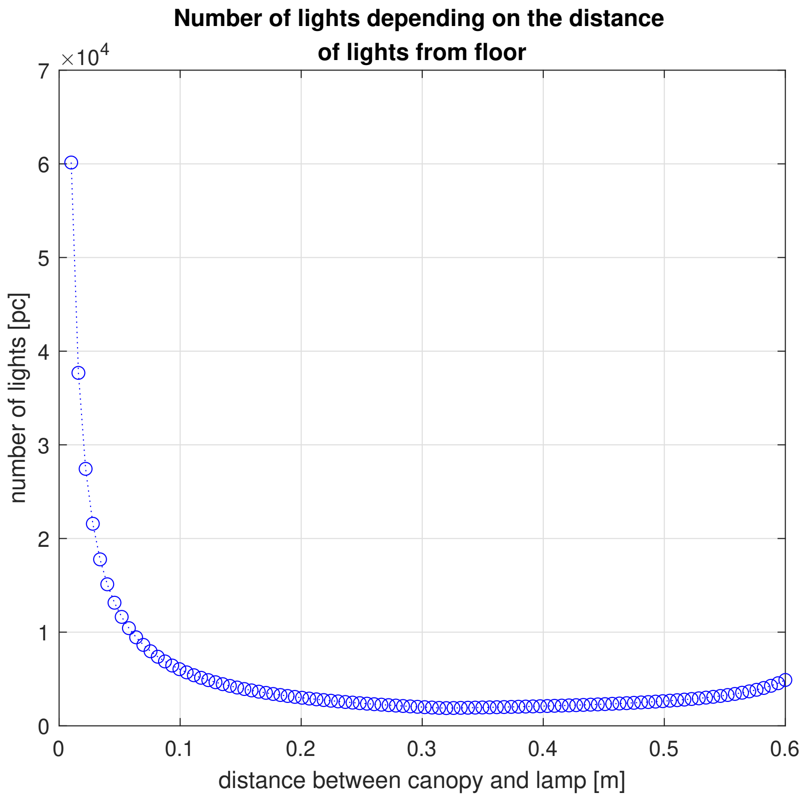

Figure 2.7: The number of lights depending on the distance.The graph is calculated for DLI = 9 $\frac{\text{mol}}{\text{m}^2\text{d}}$, lighting time 16 hours, PPF of 250 $\frac{\mu\text{mol}}{\text{s}}$ and cultivation area of size 2500 m^2.

2.4 Energy consumption

Energy consumption x_i in MWh in one hour $i \in \{1, 2, ..T\}$ is

$$x_i = N_l P_n 10^{-6} n_i y_i, \tag{2.5}$$

where N_l is the number of light,
$P_n 10^{-6}$ is nominal power of one light in watts converted to megawatts,
n_i is the light intensity at an hour i and
$y_i \in \langle 0, 1 \rangle$ is the amount of time that the light is on during an hour i.
Nominal power $P_n \in \mathbb{R}^+$ in watts is the consumed power under specific operating conditions stated in the light datasheets. The calculation of light intensity n depends, whether the number of lights N_l is estimated or specified by an expert. If the number

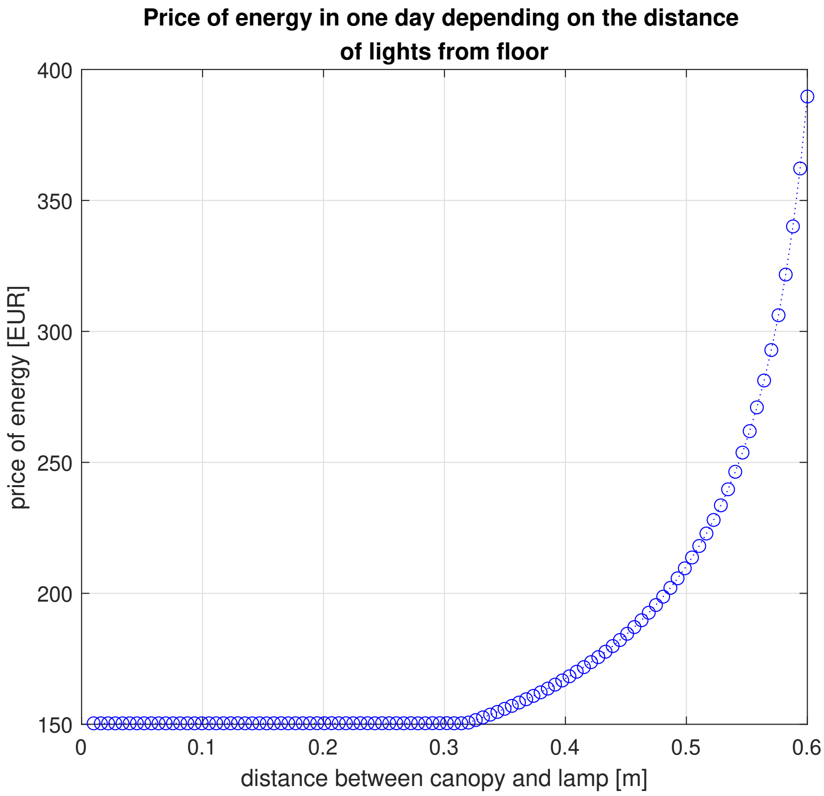

Figure 2.8: The costs of energy for one day based on the distance between light and the canopy. The graph is calculated for DLI = 9 $\frac{\text{mol}}{\text{m}^2\text{d}}$, lighting time 16 hours, PPF of 250 $\frac{\mu\text{mol}}{\text{s}}$ and cultivation area of size 2500 m^2.

of lights is not known, the calculation of intensity is described in Section 2.3.2. If there is known amount of light sources and their maximal $\text{PPFD}_{\text{max}}^{\text{lights}}$, that one light emits. Intensity is then $n = \frac{\text{PPFD}}{\text{PPFD}_{\text{max}}^{\text{lights}}}$, where PPFD is actual amount of moles and $\text{PPFD}_{\text{max}}^{\text{lights}}$ is maximal amount of moles that falls on the same surface.

The results of calculations of the number of lights and one day energy consumption are shown in Table 2.3. Lights from the same datasheet are used for comparison [21]. Prices of consumed energy $z \in \mathbb{R}$ in one day are based on pricing of energy from Nord Pool Day-ahead prices $\mathbf{c}^T$ from the Netherlands on 2023-09-18 (cells D3 till D26 in datasheet day ahead prices `day ahead prices example.xlsx`). This dataset is chosen randomly just to show a comparison of the energy prices $\mathbf{c}^T$ for one day with different lights. The pricing is based on the constant strategy described in the Chapter 3.

Table 2.3: Comparison of the number of lights N_l and energy consumption z during one day for different Food Autonomy linear top lights. All the lights are 1.2 m long. The values are for an area of 100 by 25 meters with constant lighting for 18 hours, DLI $= 9\frac{\text{mol}}{\text{m}^2\text{d}}$, distance between the light and the soil $d = 0.15$ and beam width $b_w = 150°$.

light	PPF $[\frac{\mu\text{mol}}{\text{s}}]$	P_n [W]	N_l [pcs]	z [EUR]
fix or dimmable	96	32	6472	104
fix or dimmable	145	50	3513	88
4-channel	148	51	3425	87
4-channel	200	71	2405	85
6-channel	150	65	3368	110

2.5 Relative light degradation

Light producers state in their datasheets the light lifetime using "LB" notation, where B defines the percentage of how many lights do not have L percentage of luminous flux from the initial value. For example, L90B10(40,000) means that 10 % of lights do not achieve 90 % of the initial luminous flux at T =40,000 hours [22]. For clarification, luminous flux $\Phi \in \mathbb{R}_0^+$ is the measurement of the perceived amount of light by a human eye, and it is measured in lumens. However, the amount of light that the source spread is measured in PPF. There does not exist any straight relationship between luminous flux Φ and PPF, because the human eye perceives some colours as being brighter than others. Also, the "LB" information in datasheets is for all their LED lights independent on colour.

The degradation of light can be measured under different conditions. The norm LM-80 was created by the Illuminating Engineering Society with Philips Lumileds [23] to measure the degradation of different lights. For the process of measuring light degradation, see [24]. The lifetime estimation based on LM-80 can be done more accurately than in this thesis and can almost always predict longer LED lifespan [25]. However, consumers do not have access to LED lights data, so this thesis will be using only the "LB" information. Curve fitting is based on the TM-21 norm [26, 25, 27]. The curve fitting that takes into account all the degradation mechanisms in a LED package can be written with boundary condition in the form

$$\begin{cases} \Phi(t) = \beta e^{(-\alpha t)} \\ \Phi(0) = 1 \\ \Phi(T) = L \end{cases} \tag{2.6}$$

where β is a constant equal one for relative degradation,
α is constant decay rate,

t is time in hours,
T and L are parameters from the LB.
The α is also expressed as

$$\alpha = \frac{\ln(\Phi(t))}{-t}. \tag{2.7}$$

"LB" information for different LED lights is shown in Table 2.4 and their light degradation in Figure 2.9. The degradation curves are fitted for constant lighting strategy with 100 % of light intensity. In Table 2.4 are shown lights with a 'B' percentage equal to 50 %. Using different lights, such as those with L90B20(54000) would pose a problem, since the α in 2.6 is the same as for L90B50(54000). To distinguish the degradation, the guaranteed amount of is introduced in 2.5.2.

Table 2.4: Light degradation parameters from Food Autonomy, Philips and Cool Grow datasheets.

producer	lifespan
Food Autonomy	L90B50(54000)
Philips	L95B50(36000)
CoolGrow	L90B50(50 000)

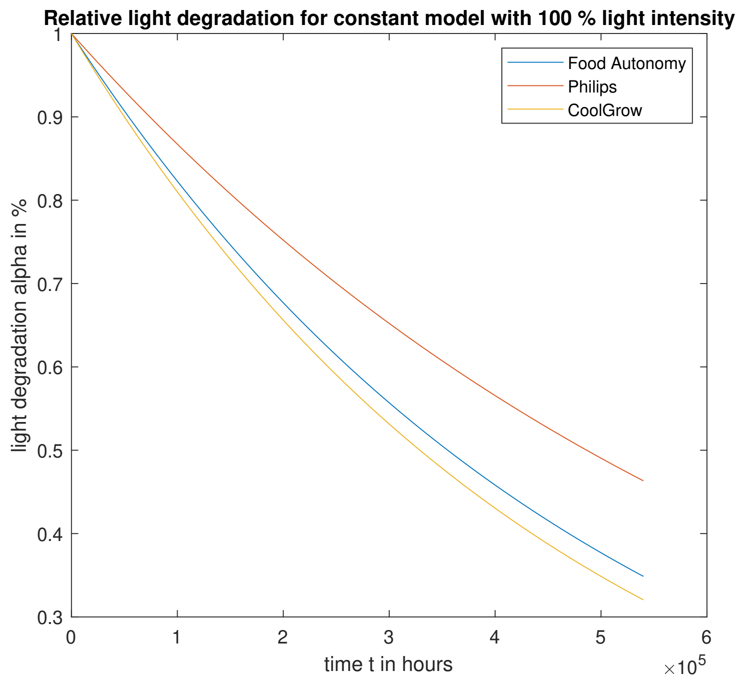

Figure 2.9: Relative light degradation for the constant strategy.

2.5.1 Degradation based on models

Degradation for the constant and the dynamic lighting strategies will be introduced in the following chapter. The constant lighting strategy has the same intensity for the whole lighting time. On the other hand, the dynamic strategy uses different light intensities for each hour. Figure 2.10 shows examples of the constant and dynamic strategies.

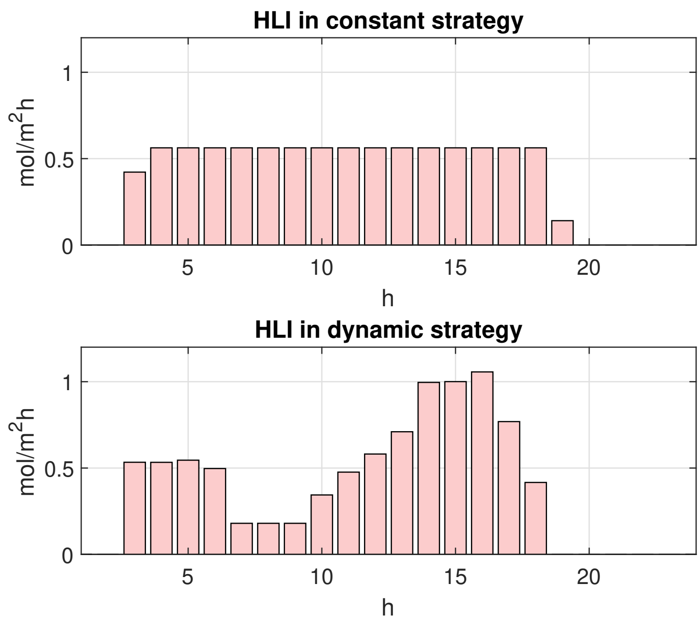

Figure 2.10: Example of hourly light integral (HLI) measured in $\frac{\mathrm{mol}}{\mathrm{m^2h}}$ for the constant and the dynamic strategy.

The parameter α in 2.6 corresponds to the temperature of the P-N junction inside the LED lamp. The exponential function correspond to the situation that the light degrades faster for higher temperature (and also intensity) and slower for lower temperature (lower light intensity). The degradation models are for both strategies further developed based on this idea.

The constant strategy $\Phi_C(t)$ and the dynamic strategy $\Phi_D(t)$ are constructed by parts of $\Phi(t)$. The degradation functions will be compounded by the differences of $(\Phi(t_i) - \Phi(t_i + 1))$ for all intensities $i \in \mathbf{n}$, where $\mathbf{n}$ is the vector of light intensities during lighting time period. The values of $\Phi(t_i)$ are from calculated degradation curve 2.6. The light intensity decay for the constant strategy is described by difference equation

$$\begin{cases} \Phi_C(t) = \Phi_C(t-1) - (\Phi(k) - \Phi(k+1)) & \text{for some } k \\ \Phi_C(0) = 1, \end{cases}$$

where $\Phi_C(t)$ is the intensity of light for constant strategy at time t and
The following steps need to be done to calculate the difference $(\Phi(k) - \Phi(k+1))$. The value of $\Phi(k)$ is known. To calculate $\Phi(k+1)$, α is expressed from 2.7 based on "LB" information. Second, the $t_i = \frac{\ln(\Phi(t_i))}{-\alpha}$ is expressed. Last, $\Phi(t+1)$ is calculated from 2.6 at time $t_i + 1$.
The dynamic strategy is also defined using a recurrence relation

$$\begin{cases} \Phi_D(t) = \Phi_D(t-1) - (\Phi(k_i) - \Phi(k_i+1)) \\ \Phi_D(0) = 1. \end{cases}$$

In the dynamic strategy, the light intensity is different each hour. This means that for each hour, the decay $(\Phi(k_i) - \Phi(k_i+1))$ for $\forall k_i$ is calculated separately. Comparison of the light decay for constant and dynamic strategy is in Figure 2.11. The degradation in 2.11 corresponds to strategies in Figure 2.10.

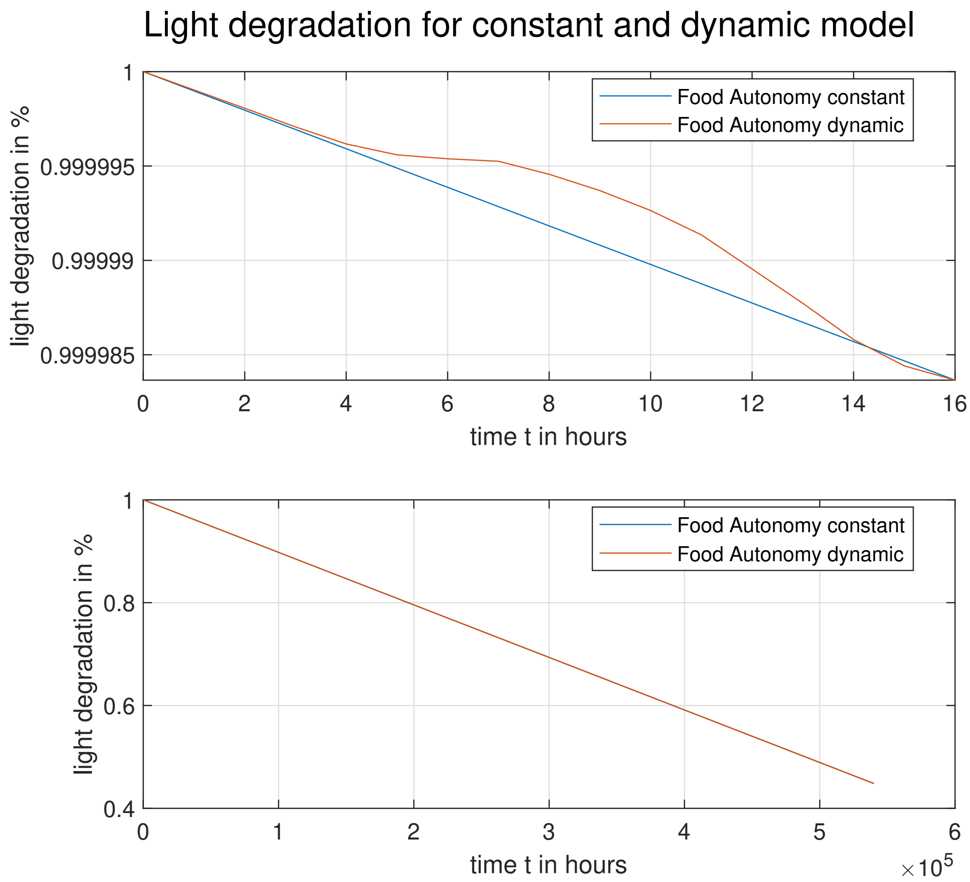

Figure 2.11: Comparison of the light degradation for continuous and dynamic strategy from Figure 2.10. The upper graph is for one day of lighting. The lower graph is for 54,000 hours of the one-day model.

2.5.2 Guaranteed amount of light

The B parameter is not involved in the last section concerned with light degradation, because most of the LED light has the B parameter equal to 50 %. For the ones that do not, the guaranteed amount of light approach is used. Two different lights L90B50(50000) and L90B40(50000) has the same the degradation curve, even if they differ in B. The guaranteed amount of light is based assumption that the number of LED lights equal to B percentage do not light at all, and $1 - B$ % of light are illuminating exactly at L value. From equation 2.6, where $\beta = 1$, α can be expressed as

$$\alpha = -\frac{\ln(\Phi(t)(1 - B))}{t},$$

where t is fixed time. Information about intensity and failure at more times, some statistic coefficient (expected value, variance, ...) or bathtub curve of failure would be needed to do more precise estimations.

2.6 IP class

Because the lights are in places with high humidity, and they get in contact with water, some lights have specified water protections. The standard IEC 62 262 describes degrees of contact and water protection. The contact protection class is the first number of IP and the second number is the degree of protection against water. The classes of water protection are in Table 2.5.

Table 2.5: Water protection classes of IP [28].

0	No protection
1	Protection against diagonally (up to 15°) dripping water
2	Protection against falling spray water up to 60° from the vertical
3	Protection against spray water on all sides
4	Protection against spray water on all sides
5	Protection against water jets (nozzle) from any angle
6	Protection against powerful water jets (flooding)
7	Protection against temporary immersion
8	Protection against continuous immersion
9	Protection against high-pressure water

2.7 Fixed or movable lights

Usually, IVFs have dimmable lights at a fixed distance from crops. Another approach to save money is to have lights with fixed intensity but with movable light fixtures.

It means that the lamps can be moved farther or closer to the crops. This approach is used in greenhouses as well as in IVFs [29, 30].

3 Methodology of plant growth considering day-ahead energy prices

This chapter introduces the types of energy contracts based on price rates and possible purchasing strategies. One of these strategies is to buy electricity on the day-ahead energy market. Based on the knowledge that the prices are from the day-ahead energy market, various lighting strategies are proposed to minimise electrical energy costs in one day. First, the lighting strategies and their parameters are introduced. Second, the mathematical definition of each strategy is provided and last, the algorithm for finding the minimal costs is stated. The end of this chapter introduces implementation of lighting strategies in Industrial Edge applications from company Siemens.

3.1 Energy contracts

Electrical energy can be traded on the exchange market or directly between supplier and buyer. Private agreements between two parties can have two types of rates

- fixed rates contract

  Customers either pay a set price for 1 MWh, regardless of energy market price fluctuations, or they agreed different prices tariffs during the day based on peak loads. Peak loads, also called peak demands, represent the highest demand on electrical power over a specified time in a day. The advantage of fixed rate contracts is that customers can easily calculate energy costs, and it protects them from rising prices.

- variable rate contract

  Payments are determined on the basis of the actual energy price. These tariffs are commonly used when anticipating lower prices or considering a change in a supplier, as they often do not have exit fees.

Electricity is traded on the exchange market in Europe by

- forward contracts

  Electrical energy, similarly to stock, oil, or gas, can be traded using derivatives. Derivatives such as futures and forwards obligate to buy or sell a specific volume in specified time for an agreed price. The difference between futures and forwards is that futures are standardised, traded on public futures exchanges, and are assured to have a low risk of not fulfilling the obligation. Meanwhile, forwards are customised and traded privately between two parties. The advantage of derivative contracts is the possibility to hedge against risk. Forwards are agreed for longer period than 1 day.

- spot contracts

  Spot contracts are executed on a day-ahead or intraday basis. The day-ahead market offers to sell or buy energy one day before the operating day.

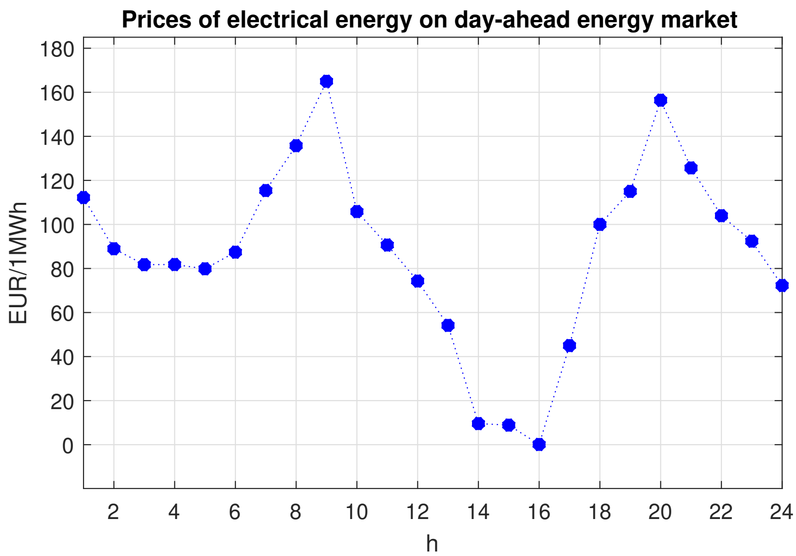

Figure 3.1: Example of prices on day-ahead energy market with electricity. Prices are for 2023-09-18 in the Netherlands from Nord Pool market [31].

3.1.1 Day-ahead energy

This thesis focuses on spot contracts, particularly on day-ahead energy market with electricity. A day before the operating day, 24 prices are published, with each price corresponding to one hour in the day. These market prices, typically set per 1 MWh (megawatt-hour), can be either positive or negative. Negative prices tend to occur during periods of low demand and an abundant energy supply. Negative prices are more likely to occur on Sundays, holidays or during the night [32]. Day-ahead electrical energy can be traded through the European Power Exchange, Nord Pool or other platforms. The Figure 3.1 depicts an example of day-ahead prices. It can be observed that the prices are the highest around 9 a.m. and 8 p.m. and lowest between 14-16 p.m. The possible causes of high prices could be high energy demand of households, meanwhile low prices during day could be caused by high supply of renewable energy.

3.2 Introduction to lighting strategies

Lighting strategies could vary in:

- colour: spectral emission of the LED lamp
- intensity: amount of PPF that the light radiates
- light time: sum of the whole time the light is on
- light cycles: cycles with alternating light and dark period
- day length: time period to optimise (mostly 24 hours)

This thesis will focus on minimising cost based on light intensity, light cycles and day length. The light intensity can be either:

- constant

  Constant strategy means that the same light intensity (PPFD) is held constant during the whole given period of time. This strategy can be applied with all types of energy contracts. It is considered the easiest strategy to control, as it keeps the light intensity constant throughout the entire lighting period.

- dynamic

  Dynamic lighting strategy is based on changing the light intensity each hour. The lights have to be either dimming or movable. The dynamic approach is either cheaper or costs the same as the constant strategy if only the daily consumed energy is considered. This is because during periods of high energy prices, the supply of energy is low, and conversely, during times of low prices, the supply is high. The dynamic strategy is the same as the constant in case when all the 24 prices are the same. This strategy is used with variable rates or spot contracts. The effect of fluctuating light intensity on plant growth is still in the research phase.

There are two types of light cycles:

- Continuous lighting without any pauses with light off.
- Intermittent light with multiple light cycles during the optimising period. Between each lighting period is time with light off.

This study will combine the light intensity and the light cycles into these four types: continuous constant, continuous dynamic, intermittent constant, intermittent dynamic lighting.

3.3 Lists of parameters

The lists below clearly specify the output parameters, input parameters and their units. Input vector from the day-ahead energy market

$\mathbf{c}$	vector of prices	currency (€, \$, ...)

Input parameters that represent conditions for specific crop

DLI	daily light integral	$\frac{\text{mol}}{\text{m}^2\text{d}}$
PPFD_{min}	minimal amount of moles that crop needs	$\frac{\mu\text{mol}}{\text{m}^2\text{s}}$
PPFD_{max}	maximal amount of moles that crop absorb before damage	$\frac{\mu\text{mol}}{\text{m}^2\text{s}}$
$M_{\frac{1}{4}}$	maximal volume of energy supplied in quarter of an hour	MWh
p_s	number of hours that lights are off from the start of the day	h
p_n	number of hours that lights are off at the end of the day	h
h	number of hours that the lights are on in one cycle (con. strat.)	h
$\mathbf{v}$	vector that describes light cycles (intermittent strategies)	h

Input parameters from light datasheets

P_n	nominal power	W

There are now two possible parameters. One is used when there are custom-made lights for a specific IVF. The other occurs when the number of lights is estimated. For the first possibility

N_l	number of lights	pc
$\text{PPFD}_{\text{max}}^{\text{lights}}$	maximal PPFD that the light emits for given distance r	$\frac{\mu\text{mol}}{\text{m}^2\text{s}}$

In case, where the number of lights is estimated

C_l	cultivation area length	m
C_w	cultivation area width	m
r	distance between light and canopy	m
l_l	length of linear light	m
b_w	angle of light emission	DEG
PPF	amount of moles that light emits	$\frac{\mu\text{mol}}{\text{s}}$

Output parameters

$\mathbf{x}$	vector of bought volume of energy	MWh
$\mathbf{y}$	vector of ratios specifying the length of time the light was on in each hour	h
z	paid price for one day	currency

3.4 Problem definition

The main goal of IVF optimisation is to minimise spending on electrical energy, which is represented in 3.1 by the objective function z. For each day, 24 prices of energy are available, each for one hour of said day. This thesis allows optimising over any time period $T \in \mathbb{N}$, because some IVFs use different lengths of growth cycles. Prices are represented as a vector $\mathbf{c} = [c_1, c_2, \ldots, c_T]$, where $c_i \in \mathbb{R}$. The goal is to determine the optimal volume $\mathbf{x} = [x_1, x_2, \ldots, x_T]$ of energy in MWh. Each volume x_i corresponds to the price c_i. The minimisation problem is formulated as:

$$\min_x z = \mathbf{c}^T \mathbf{x} \tag{3.1}$$

$$\forall i \in \{1, 2, \ldots, T\} : x_i \geq 0$$

$$x \in \mathbb{R}^T$$

The definition of this problem is not final, because constraints about the maximal power consumption, the minimal and maximal PPFD constraints and the definition of the specific strategy are missing.

Maximal power consumption

Maximal power consumption $M_{\frac{1}{4}}$ is measured by energy supplier in Europe quarterly in one hour [33, 34]. For the constant strategy where light intensity remains the same, only the first hour is checked for maximal power consumption. In the first hour $j = \min\{i|x_i > 0\}$ is a possibility that the light will be switched on during or at full hour. There exists a vector $\mathbf{y} = [y_1, y_2, .., y_T]$, where $y_i \in \langle 0, 1\rangle$ expresses the percentage of time, that LED lights are on during one hour. E.g., if the lights can be on each quarter of an hour, the $y_i \in \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$. The first constraint for constant strategy is then defined as $j = \min\{i|x_i > 0\} : x_j \leq 4M_{\frac{1}{4}} y_j$. This problem is generalised for the dynamic strategy, where all the values are checked. Therefore

$$\forall j \in 1, 2, ...T : x_j \leq 4M_{\frac{1}{4}} y_j. \tag{3.2}$$

PPFD

The next constraint is based on the IVF's requirements. High light intensity could lead to crop damage by photoinhibition (decrease in photochemical efficiency caused by intense radiation). On the other hand, low intensity could lead to nutrition imbalances due to reduced nutrient uptake. Lack of chlorophyll production causes that crops are more susceptible to diseases and have yellow leaves due to low light intensity. The PPFD constraint with minimal $\mathrm{PPFD_{min}}$ and maximal $\mathrm{PPFD_{max}}$ are checked by a given DLI and the amount of hours $h \in \mathbb{N}$, that the light is on, computed as $h = \sum_{i=1}^{T} \mathbf{y_i}$. The DLI is a given constant. The relationship between PPFD and DLI is given above, see 2.1. Again, for the constant lighting strategy, only one constraint needs to be checked

$$\mathrm{PPFD_{min}} \frac{3600}{10^6} h \leq \mathrm{DLI}, \tag{3.3}$$

$$\mathrm{PPFD_{max}} \frac{3600}{10^6} h \geq \mathrm{DLI}. \tag{3.4}$$

For the dynamic strategy, each hour is checked separately by hourly light integral (HLI). The relationship between PPFD and HLI is $\mathrm{HLI} = \mathrm{PPFD} 3.6^{-3}$, see 2.1. To remind

$$\sum_{j=1}^{T} \mathrm{HLI}_j = \mathrm{DLI}.$$

The constraint for minimal and maximal PPFD are checked as

$$y_j \mathrm{HLI_{min}} \leq \mathrm{HLI}_j,$$

$$y_j \mathrm{HLI_{max}} \geq \mathrm{HLI}_j.$$

Already mentioned maximal energy consumption and the maximal PPFD constraint can be checked together. The new constraint is called maximal volume constraint. The maximal energy consumption is represented by a vector $\mathbf{V}_{\max}^{\text{vol}} = 4M_{\frac{1}{4}}\mathbf{y}$. The maximal PPFD constrains is calculated from $\text{PPFD}_{\max}$ or $\text{HLI}_{\max}$ to volume $\mathbf{V}_{\max}^{\text{PPFD}}$. The relationship between bought volume and light intensity is described in equation 2.5 as

$$\mathbf{V}_{\max}^{\text{PPFD}} = N_l P_n 10^{-6} \mathbf{n} \odot \mathbf{y}, \tag{3.5}$$

where $\odot$ is elementwise multiplication of two vectors, also called Hadamard product. The maximal volume constraint is the lowest of the previously counted maxims

$$\mathbf{V}_{\max} = \min(\mathbf{V}_{\max}^{\text{PPFD}}, \mathbf{V}_{\max}^{\text{vol}}). \tag{3.6}$$

The constraint is

$$\forall x_i; i \in \{1, 2, ..., T\} : x_i \leq V_{\max}.$$

Constraint of light off

The optimisation strategies will find the optimal time to have a light on during one day. The light has to be on, e.g., for 16 hours. There is possibility that one day the optimalizator found the optimal lighting time at the end of the day and next day at the start of the day. It means that the light would be on for 36 hours continuously. From that reason, pause from start of the day p_s or pause at the end p_e are defined. The constraint is

$$p_s + p_e < T - h \tag{3.7}$$

$$p_s \in \{0, 1, 2, T - 1\}$$

$$p_e \in \{0, 1, 2, T - 1\}$$

For y_k, x_k, HLI_k is true that

$$\forall k \in \{1, 2, ..T\} : y_k = 0 \Leftrightarrow x_k = 0 \Leftrightarrow \text{HLI}_k = 0$$

and also

$$\forall k \in \{1, 2, ..T\} : y_k > 0 \Leftrightarrow x_k > 0 \Leftrightarrow \text{HLI}_k > 0.$$

3.5 Constant continuous lighting

The constant continuous lighting strategy has only one light cycle where the light intensity remains the same over the whole given period of time. An example of this strategy could be seen in Figure 3.2. The top subfigure depicts the day ahead energy prices. The middle subfigure shows the hourly light integral HLI in $\frac{\mu\text{mol}}{\text{m}^2\text{h}}$ in each hour. The subfigure can lead to a misleading idea that the light intensity during the third hour $h = 3$ and the nineteenth hour $h = 19$ is different. However, the light's brightness stays the same during the whole period. The light is on from 2:15 to 19:15. The last subfigure shows dependence of bought volume on energy on hour.

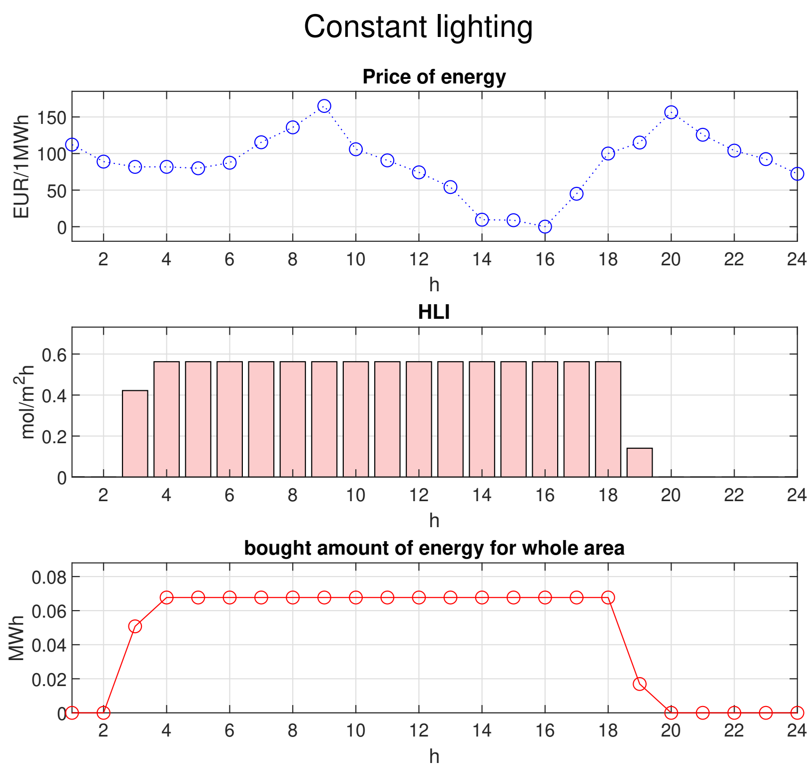

Figure 3.2: Continuous lighting with 16 hours of light on. The model includes a constraint of 2 hours of the light being turned off from the start of the day and 2 from the end. The lighting can be switched on or off each quarter of an hour. The 3rd hour, the light is on for 3/4 of the said hour and for the 19th hour, the light is on only for 1/4.

3.5.1 Model definition

The lighting time with minimal costs is found with the respect to the strategy. For the constant continuous strategy, the light is on for h hours. It is break down to $h = s_2 - s_1$. $\langle s_1, s_2 \rangle$ represents the time span when the light is on. For $s_1, s_2 \in \langle 0, T \rangle$ is bounded by

$$p_s \leq s_1 < s_2 \leq T - p_e. \tag{3.8}$$

For the vector of ratios for how long the light is on $\mathbf{y}$ in hour i, the constant continuous strategy can be rewritten to form

$$\forall l \in \{\lceil s_1 \rceil + 1, \lceil s_1 \rceil + 2, ..., \lfloor s_2 \rfloor\} : y_l = 1 \tag{3.9}$$

$$\begin{aligned} y_{\lceil s_1 \rceil} &\in \langle 0, 1) \\ y_{\lceil s_2 \rceil} &= 1 - y_{s_1}. \end{aligned} \tag{3.10}$$

The $\lceil g \rceil$ and $\lfloor g \rfloor$ are the ceiling and floor functions of a number g. The same also applies for bough volume of energy

$$\forall m, n \in \{\lceil s_1 \rceil + 1, \lceil s_1 \rceil + 2, ..., \lfloor s_2 \rfloor\} : x_m = x_n$$

$$\begin{aligned} x_{\lceil s_1 \rceil} &= y_{\lceil s_1 \rceil} x_m \\ x_{\lceil s_2 \rceil} &= x_m - x_{\lceil s_1 \rceil}. \end{aligned} \tag{3.11}$$

3.5.2 Setting the conditions

First, parameters regarding crop and lighting conditions are set in `model_code.m`. The parameters are following

- customize - Firstly, the IVF has to choose between the continuous and the intermittent strategy. The value 1 corresponds to the continuous and the value 2 to the intermittent strategy.

- hoursLightOn - Then the IVF states for the continuous strategy, the number of hours when the light on h.

- vec_hours - In the case of the intermittent strategy, a vector of hours that the light is on and off is set. The dimension of the vector corresponds to the dimension of the vector of day-ahead prices $\mathbf{c}$. The vector contains only ones and zeros, where ones represent that the light is on and zeros that the light is off. The first element is always set to one.

- pausStartDay and pausEndDay - Because the optimisation method will be finding the optimal price among all time periods and some crops are 'short day plants', it has to be ensured that the light on period will not end at the end of the day and the new period will not start at the start of the new day. This means that the quantities such as pause from the start of the day $p_s \in \mathbb{N}_0$ and the pause from the end of the day $p_e \in \mathbb{N}_0$ has to also be stated. The pauses are given in hours.

- window - There is also a possibility to change the switching window $w \in \mathbb{N}$, which specifies how many times per hour it is possible to start a lighting period, e.g., 4 means the possibility to turn the light on every quarter of an hour, 6 every 10 minutes and so on. The higher the value of w is, the lower (or equal) $\min_x z$ is found.
- DLI - Daily light integral.
- lower_ppfd and upper_ppfd - The lowest $\mathrm{PPFD}_{\mathrm{min}}$ and the highest $\mathrm{PPFD}_{\mathrm{max}}$ that a crop can have during a light on period.
- upper_bound_quater - Because the supply of energy could be limited by the energy network or the supplier, the maximum amount of energy consumption in each quarter of an hour is also stated.
- power - Nominal power in watts.
- know_lights - The next step is to choose between either known, or estimated number of lights. The value 1 signifies known the number of lights, while 0 means that the number of lights is not known.
- areaLength, areaWidth, lightLength, lightToGround, beamwidth, photonFlux - In the case that the number of lights is unknown the area length, width, light length, distance between light and canopy, light beam width and light photon flux PPF has to be stated.
- numberOfLights and max_ppfd - In case where the lights are known, only the number of lights and the maximal PPFD that light can emit PPFD_l is stated.

Source code 3.1: Setting of parametrs

```
%set 1 for continuous lighting , set 2 for custom made
customize = 1;

%customize = 1 - only whole hours 14,15,16,... hours of
    light
hoursLightOn = 16;
%customize = 2 - vector always start with 1
%1 mean light is on, 0 light is off
vec_hours
    =[1,1,1,0,0,1,1,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0]';

% LIGHTING TIME CONDITIONS
% switching period: 4 is for 15 min, 6 would be for 10,..
```

```
window = 4;
pausStartDay = 0;  % in hours
pausEndDay =0;     % in hours

% CROP CONDITIONS - daily light integral and min and max
   PPFD
DLI = 12;
lower_ppfd = 150;
upper_ppfd = 300;

% MAXIMUM OF ENERGY CONSUMPTION IN 1/4 OF HOUR
upper_bound_quater = 1;

% LIGHT CONDITIONS
power=32;

know_lights = 0;
% the number of lights is not known
if know_lights ==0
    areaLength = 100;
    areaWidth = 25;
    lightToGround = 0.1;
    lightLength = 1.201;
    beamwidth = 126; %in degrees
    photonFlux=97;

% the number of lights is known
elseif know_lights ==1
    numberOfLights = 5000;
    % maximal PPFD on surface
    max_ppfd = 557;
    % upper constraint for max PPFD
    upper_ppfd = min(upper_ppfd, max_ppfd);
end
```

3.5.3 The algorithm of finding the $\min_{\mathbf{x}} z = \mathbf{c}^T\mathbf{x}$

The algorithm of finding the minimum has the following steps:

1. step - Firstly, the constraints for maximal and minimal amount of PPFD in 3.3 and 3.4 are checked. Because the intensity will be the same for every found minimum, the constraint can be checked only once at the start. This constraint checks that the given DLI correspond with the PPFD bounds.

Then the first vector of hours during which the lights are on is created. The vector has first h values equal one and rest are zeros, e.g. for $h = 3$ the vector is $\mathbf{y} = [1, 1, 1, 0, ..0, 0]$. In the case where p_s is more than 0, the first 1 starts at the $p_s + 1$ position. Then, the initial value of $\mathbf{c}^T\mathbf{y}$ is calculated.

Repeated steps - The constant lighting strategy is computed in k number of steps where $k = w(T - h - p_s - p_e) - 1$. In each step, the vector of light hours $\mathbf{y}$ is shifted by the chosen light window $\frac{1}{w}$, e.g. for $w = 4$ second step vector equals $\mathbf{y} = [0.75, 1, 1, ...1, 0.25, 0...0, 0]$. The sum of y is the same, it is equal to the initial number of lighting hours h. The shifted vector is compared with the last minimum $\mathbf{c}^T\mathbf{y}'$ in each step. In the case where $\mathbf{c}^T\mathbf{y}' > \mathbf{c}^T\mathbf{y}$, the $\mathbf{y}$ is taken as the new minimum. If $\mathbf{c}^T\mathbf{y}' < \mathbf{c}^T\mathbf{y}$ the $\mathbf{y}'$ stays as the minimal value and it is compared in next steps.

End- This type of strategy is computed by splitting the DLI uniformly over the time period when the light is on. First, the hourly light integral is calculated as $\mathbf{HLI} = \mathbf{y}\frac{\mathrm{DLI}}{h}$, for the found minimal $\mathbf{y}$. Then the volume $\mathbf{x}$ is computed using 3.5. The constraint for maximal volume consumption 3.2 is checked. If the constraint holds, the result of the loss function is $\min_{\mathbf{x}} z = (\mathbf{c}^T\mathbf{x})$. If not, the problem does not have a feasible solution.

3.5.4 Extension of strategies

The length of optimising period does not have to be 24 hours. One study proposes light-cycles strategies with light/dark period equals to 16/8 h, 16/4 h or 16/2 h [35]. Significant growth of crop was measured in 16/2 period. This leads to create a lighting strategy with day length shorter than 24 hours. There is a possibility to use $\mathbf{y} \in [y_1, y_2, ..., y_T]$ with any length $T \in \mathbb{N}$, where $T > h$.

3.6 Dynamic continuous lighting

Amount of light is never stable in the nature due to clouds. This idea inspire the concept of dynamic strategy. The goal is to adhere DLI, while adjusting PPFD of a lighting system each hour throughout the day. Although the dynamic lighting approach is not directly derived from existing research papers, lighting companies are creating dimmable lights for dynamic lighting solutions compatible with renewable energy systems, see [36]. The example of dynamic lighting strategy is shown in Figure 3.3.

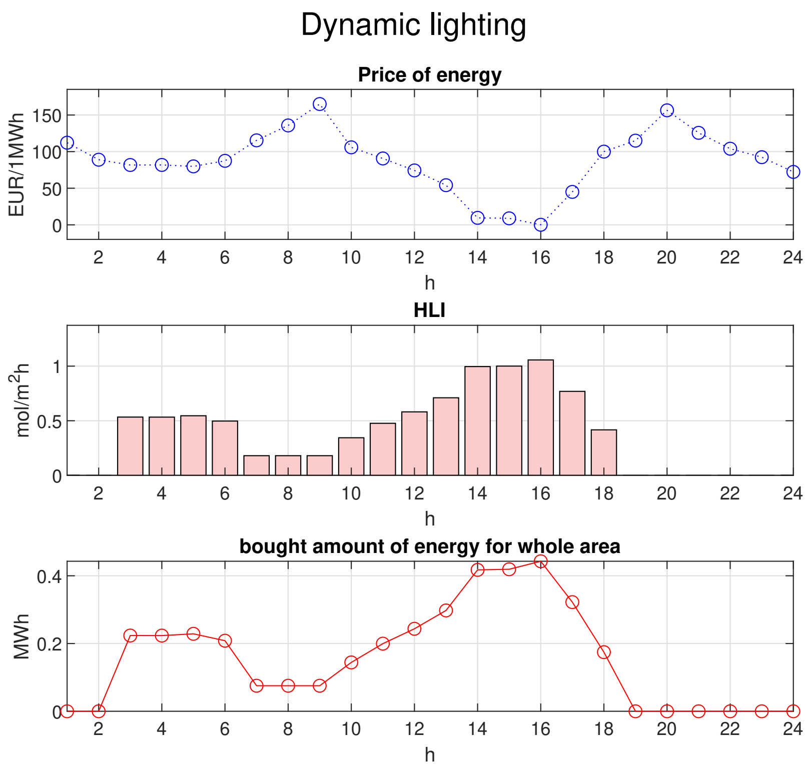

Figure 3.3: Dynamic lighting with 16 hours of light on. Model is with condition of 2 hours of light off from the start of the day and 2 from the end. The lighting window is set for 4. The light is on each hour from 3 to 18 o'clock included.

3.6.1 Model definition

The minimum is found with the respect to the strategy. Again, the time span $\langle s_1, s_2 \rangle$ when the light is on is defined as

$$p_s \leq s_1 < s_2 \leq T - p_e.$$

For the vector of ratios for how long the light is on $\mathbf{y}$ in an hour i, is identical with constant continuous strategy. Just to repeat,

$$\forall l \in \{\lceil s_1 \rceil + 1, \lceil s_1 \rceil + 2, ..., \lfloor s_2 \rfloor\} : y_l = 1$$

$$y_{\lceil s_1 \rceil} \in \langle 0, 1)$$

$$y_{\lceil s_2 \rceil} = 1 - y_{s_1}.$$

For volume is true that

$$\forall l \in \{\lceil s_1 \rceil, \lceil s_1 \rceil + 2, ..., \lceil s_2 \rceil\} : x_l > 0.$$

3.6.2 Algorithm of finding the $\min_{\mathbf{x}} z = \mathbf{c}^T\mathbf{x}$

The setting of conditions remains the same as in the constant strategy. The algorithm of finding the minimal price is following:

1. step - Firstly is created vector of times during the lights are on as first h hours $\mathbf{y} = [1, 1, 1..0, 0]$. In case where $p_s > 0$, the first 1 starts at the $p_s + 1$ position. Then is calculated the expected value of first lighting period as an arithmetic average $\bar{E} = \frac{\sum \mathbf{c}^T\mathbf{y}}{\sum \mathbf{y}}$. The values for dynamic strategy are reversed $\mathbf{r} = \bar{E} - (\mathbf{c} - \bar{E})$. Because the volume can be only positive, the reversed vector is shifted in case that some of r_i is negative value $\mathbf{r} = \begin{cases} \mathbf{r} - \min(\mathbf{r}) + 10^{-3} & \min(\mathbf{r}) \leq 0 \\ \mathbf{r} & \text{otherwise} \end{cases}$. After this operation is $r_i = r_i y_i$ to assure that reverse values are positive only at specific time. The reversed vector is scaled $\mathbf{s} = \mathbf{r}\frac{\text{DLI}}{\sum \mathbf{r}}$ to have the sum equal to the given DLI. The scaled values have to also fit into the thresholds. The scaled vector is sort in descending order $\mathbf{d}$. The PPFD bound are converted to $\text{HLI}_{\text{min}} = \text{PPFD}_{\text{min}} 3.6^{-3}$. If there $\exists i \in \{T, T-1, ..., 3, 2\} : d_i < \text{HLI}_{\text{min}}$ which exceed the lower bound of HLI, the $d_i = \text{HLI}_{\text{min}}$ and overflowing part $d_i - \text{HLI}_{\text{min}}$ is shifted, so the $d_i = \text{HLI}_{\text{min}}$ and the $d_{i+1} = d_{i+1} + (d_i - \text{HLI}_{\text{min}})$. This is repeated until the $i = 2$. For the case $i = 1$ is not necessary to control the lower bound, because DLI = HLI and in the worst case is made the constant strategy. Then the number of lights and the volume $\mathbf{x}$ is calculated the same way as in constant strategy. Then the conditions for maximal energy consumption and maximal PPFD are checked. The maximal volume $\mathbf{V}_{\text{max}}$ is counted based on 3.6 and the upper condition is checked similarly as the lower bound condition. If there $\exists i \in \{1, 2, ...T\} : x_i > V_{max}$ which exceed the upper bound of volume, the $x_i = V_{max}$ and overflowing part $V_{\text{max}} - d_i$ is shifted to the $x_{i+1} = x_{i+1} + (V_{\text{max}} - d_i)$. This is repeated until the $i = T$. The vector x is resorted back to its prime positions and is counted the first minimum $\mathbf{c}^T\mathbf{x}$, which is compared in the next step.

Repeated steps - The dynamic lightening strategy is calculated also in $k = w(24 - h - p_s - p_e) - 1$ number of steps, where all steps start with shifting the intensity vector y by chosen window. Due to counting the expected value, there is created new vector of ones $o_i = \begin{cases} 1 & y_i > 0 \\ 0 & \text{otherwise} \end{cases}$ and the expected value is counted as $\bar{E} = \frac{\sum \mathbf{c}^T\mathbf{o}}{\sum \mathbf{o}}$. The algorithm continues the same way as in step 1,the values r are counted, reversed, shifted and multiplied so $\forall i : r_i = r_i y_i$. The new $\mathbf{r}$ is scaled and sorted. The bounds are not scalars, but vectors, since the first and last value of y are not the whole hours. The lower threshold is $\mathbf{HLI}_{\text{min}} = \text{HLI}_{\text{min}}\mathbf{y}$. The thresholds are sorted by the same indexing as the r. The values under and above are shifted the same way as in step one, however now are compared parts of vector $\text{HLI}^i_{\text{min}}$ with

d_i.Then the amount of lights is counted and conditions for maximal bought volume are checked. The new minimum is the min $\mathbf{c}^T\mathbf{x} = \min(\mathbf{c}^T\mathbf{x}, \mathbf{c}^T\mathbf{x}')$, where $\mathbf{x}'$ is the calculated volume in actual step.

3.7 Intermittent lighting

Intermittent lighting strategy is the opposite of the continuous lighting. In the intermittent lighting strategy, the light is switched on and off multiple times during one day. Some research papers are dedicated to comparing influence of intermittent and continuous lighting strategy [37]. Some specific intermittent strategies show that they do not have statistically different results in the development rate, the growth, and the harvest-ready size of the plants. Next, the intermittent strategy is more optimised in [38]. An example of intermittent constant lighting strategy is seen in Figure 3.4 and intermittent dynamic strategy is in Figure 3.5. Both strategies are set of 9 hours.

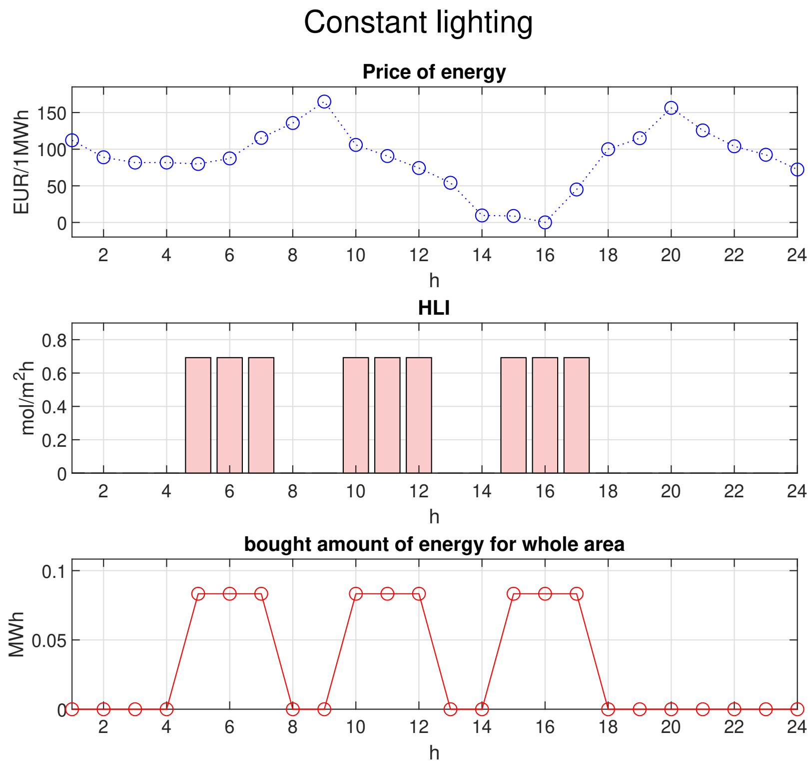

Figure 3.4: Custom constant lighting strategy with 9 hours of light. The light is on 3 times for 3 hours with two hours pauses. Model is with condition of 2 hours of light off from the start of the day and 2 from the end.

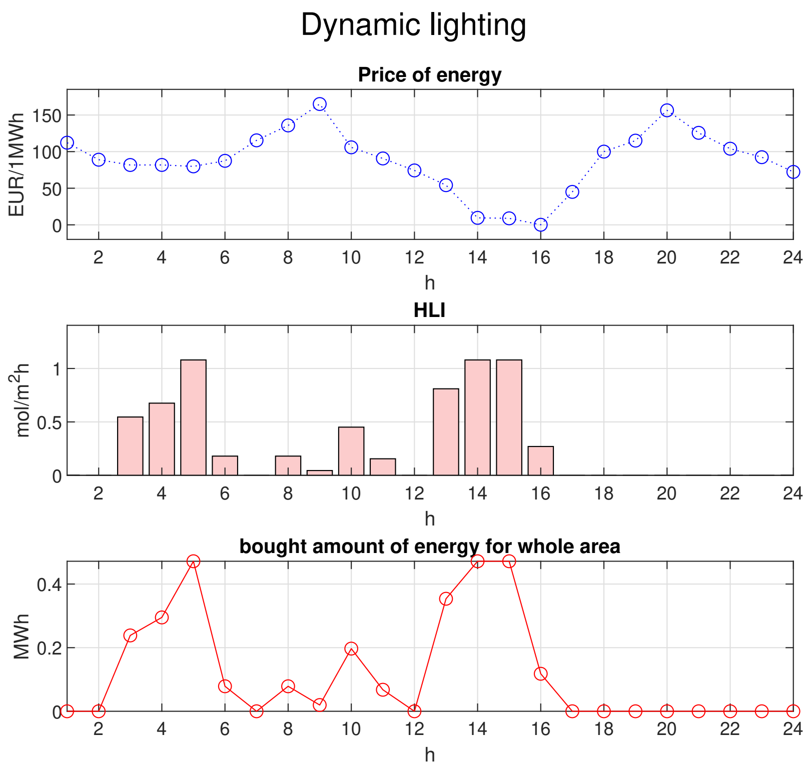

Figure 3.5: Custom dynamic lighting strategy with 9 hours of light. The light is on 3 times for 3 hours with two hours pauses. Model is with condition of 2 hours of light off from the start of the day and 2 from the end.

3.7.1 Model definition

The condition for pauses is the same as in constant and dynamic strategy in 3.8. However, h is not equal to $s_2 - s_1$, because h is split to multiple time periods. Instead $s_1 = \min\{i|y_i > 0\}$ and $s_2 = \max\{i|y_i > 0\}$.

The conditions 3.9 to 3.11, which define the ration of time that the light is on $\mathbf{y}$ and the bought volume $\mathbf{x}$, needs to be rewritten. The model have multiple periods when the light is switched on and of. The first times, when the light is switch on create a set $\mathbf{F} = \{k \mid (y_{k-1} = 0 \vee \nexists y_{k-1}) \wedge y_k > 0\}$ and set of last values $\mathbf{L} = \{l \mid (y_{l+1} = 0 \vee \nexists y_{l+1}) \wedge y_l > 0\}$, where $k \in \mathbb{N}$ and $l \in \mathbb{N}$. For the vector $\mathbf{y}$ applies that

$$\forall k \in F : y_k \in (0, 1\rangle \wedge x_k > 0$$

$$\forall l \in L, y_l \in (0, 1\rangle \wedge x_l > 0.$$

The set F and set L are indexes in increasing order, where first index is k_1, second k_2 and so on to some k_n, for the set L applies the same, the first index is index l_1, second $l_2,..l_n$. Generally applies that

$$\forall j \in \{k_j + 1, k_j + 2, ..., l_j - 2, l_j - 1\} : y_j = 1 \wedge x_j > 0,$$

where $j \in \{1, 2, .., n\}$. n is a number of first or last values. For constant strategy is also true that $\forall j \in \{k_j+1, k_j+2, ..., l_j-2, l_j-1\}$ and $\forall i \in \{k_i+1, k_i+2, ..., l_i-2, l_i-1\} : x_j = x_i$.

3.8 Linear programming

The above mentioned model definitions consist of linear objective functions, linear constrains and the determined $\mathbf{x} \in \mathbb{R}^n$. It means that the problem falls within the area of linear programming (LP). LP problems aim to minimize or maximize a linear objective function subject to equality and inequality constraints [39, 40].

Definition 3.8.1 (General definition of linear programming problem)
Let $\mathbf{A} \in \mathbb{R}^{m \times n}$ be given matrix, N is subset of columns indexes $\{1, 2, ..., n\}$ of matrix $\mathbf{A}$ and N^c is complement of N. M is subset of rows indexes $\{1, 2, ..., m\}$ of matrix $\mathbf{A}$ and M^c is complement of M. a_i are rows of $\mathbf{A}$, where $i \in \{1, 2, ..., m\}$. $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$ are given vectors. The aim is to find a vector $\mathbf{x}$ of decision variables

$$\min \ z = \mathbf{c}^T \mathbf{x},$$

subject to

$$\begin{aligned} \mathbf{a}_i^T \mathbf{x} &= b_i && \textit{for } i \in M, \\ \mathbf{a}_i^T \mathbf{x} &\geq b_i && \textit{for } i \in M^c, \\ x_j &\geq 0 && \textit{for } j \in N, \\ x_j &\in \mathbb{R} && \textit{for } j \in N^c. \end{aligned}$$

Simplex method, dual simplex method, interior point method, branch and bound method and many more methods are used to solved LP problems. The algorithms of finding the optimal time of light on in section 3.5.3 and 3.6.2 are not standard. The algorithms are implemented in applications with limited options. Proposed algorithms are simple and correspond to the application complexity.

3.9 Do the algorithms find the optimal solution?

In the case of constant strategy, it is easy to argue that the algorithms find the optimal solution. The final amount of possible prices $\mathbf{c}^T \mathbf{x}$ are compared. Moreover, intensities and volume remain the same during the whole process of finding the minimum.

It makes the definition of the strategy and algorithm explicit. On the other hand, the definition of the dynamic lighting strategy is not explicit, and various approaches to define the dynamics can be used. The one that would find the real minimum based on the definition would be the one, where the lighting is on maximum in hours with the lowest prices and on minimum in hours, when the price of energy are highest that day. Of course, all the conditions are met.

The solution of the algorithm of dynamic strategy, described in 3.6.2, is not minimal by the definition. Thanks to the finite amount of possibilities, it founds the minimal price for energy with respect to the type of algorithm. This strategy is chosen, because of lack of research about influence of dynamic strategy on crops. This strategy has lower fluctuations of intensity and copies natural sources of energy in reverse order.

3.10 Comparison of lighting strategies

The paid price for energy in one day will be compared in three different cases. In the first case is used any optimisation. The light are on each day in the last h hours at the same time. The second case is constant continuous lighting strategy, and the third case consist of dynamic continuous lighting strategy.

The cost of energy remains the same across all strategies when the price of energy per megawatt-hour is uniform throughout the day. Under these conditions, the dynamic strategy is equivalent to the constant strategy, and the timing of when the light is on or off does not impact the price. However, if the price fluctuates even once during the day, the dynamic strategy will result in a lower overall energy cost for that day. It implies that using optimisation strategies pays off in cases where the prices are not fixed. How much the strategy pays off depends on the fluctuation of the prices and the chosen lighting time in the non-optimised strategy.

In Figure 3.7 can be seen prices for one day with extreme at 4 p.m. In Figure 3.7 are graphs showing different strategies, where the non-optimised strategy is set to have lights on for the last 16 hours of the day. The light is set on in the non-optimised strategy at the wrong time, and the paid prices of the constant and dynamic strategies are around 60 % of the non-optimised strategy.

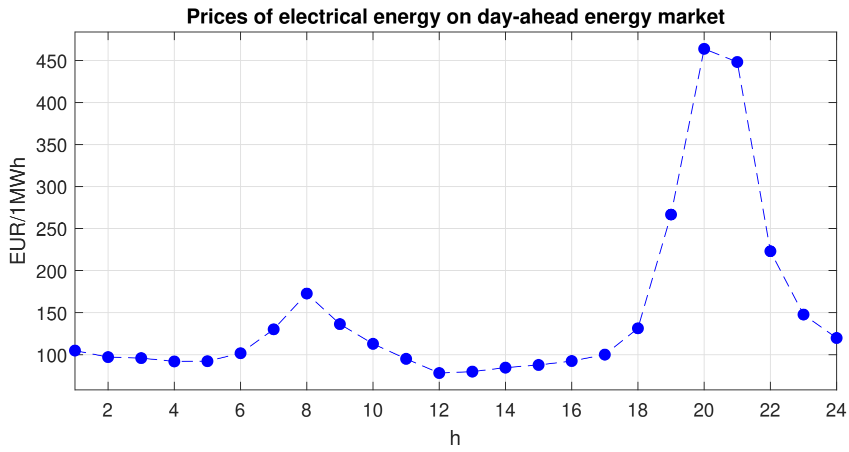

Figure 3.6: Example of prices on day-ahead energy market with electricity. Prices are for 2023-09-11 in the Netherlands from Nord Pool [31].

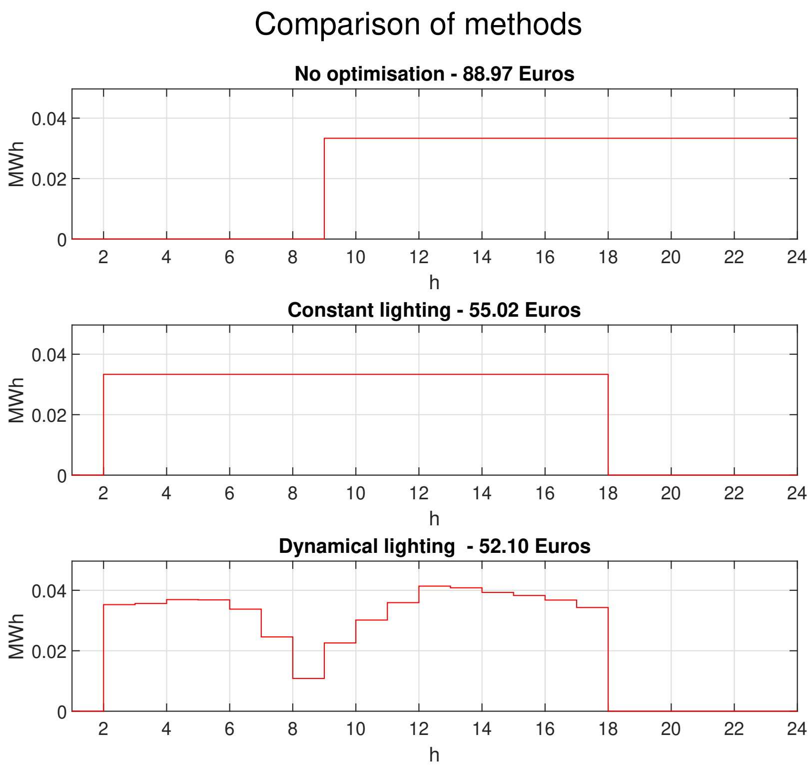

Figure 3.7: The graphs are showing the dependency of bought MWh on hours based on prices during the day in Figure 3.6. The paid price for one day is stated next to the title of each graph. The light is on continuously in all strategies for 16 hours.

3.11 Implementation in Industrial Edge applications

The algorithms of finding the minimal price for electrical energy based on lighting strategies are shown until this point in Matlab. To extend this thesis to practice, the lighting strategies are implemented in Industrial Edge applications from company Siemens. Used applications optimise constant and dynamic continuous lighting strategies for 24 input values of prices with optimisation window equal to 4.

Steps of implementation:

1. Create function in Simulink

   The Siemens application LiveTwin accepts as import any Functional Mock-Up interface application. It means that there can be imported any XML, C, C++ and other codes in ZIP file. Matlab Coder enables to either generate C or C++ code. From the Matlab code `model_code.m` is created Simulink function. In Figure 3.8 is shown the function with input parameters on the left side of a block. The block contains the function with optimalizator of lighting strategies. On the right side of the block are outputs of the function.

2. Download data from Nord Pool web page

   The second step is to get data for optimisation from a web page. Data used in this example are from Germany 2024-03-24 and are shown in Figure 3.9. The data are download and give straightly by user to the application. For a full automatization, API scratch is needed to extract data from a web page.

3. LiveTwin

   LiveTwin also called digital twin is simulation of real objects in computer to predict errors and dysfunctions in real world. LiveTwin could further focus on engagement of sensors or monitoring quality of crop. Light sensors could help to predict the light degradation and optimise the time, when it is necessary to change the LED lights. The input parameters to the LiveTwin are in the Figure 3.11. The parameters are for Humulus Lupulus in vegetative stage of growth.

4. Create data flow from LiveTwin to Energy Manager in Flow Creator

   The output data from LiveTwin are sent to Energy Manager via Flow Creator. Flow Creator is an application tool for wiring together hardware devices in runtime or in a single-click, see example 3.11.

5. Display in Energy Manager

Energy Manager is an application to monitor, optimise and visualise energy consumption and costs. In Figure 3.12 can be seen visualised priced of energy from Nord Pool energy market. In Figures 3.12 and 3.13 are visualised result of lighting strategies. In the left upper corner are prices for energy. In the right upper corner from left to right are prices for no strategy, constant and dynamic strategy for one day. In the left lower corner is bought amount of energy for that day, and in the right are shown intensities of light for that day. Red colour is for no strategy, blue for constant strategy and green for dynamic strategy.

The lighting optimisation in Energy Manger was presented in Industrial Solution Fair at Hannover from 2024-04-22 to 2024-04-26.

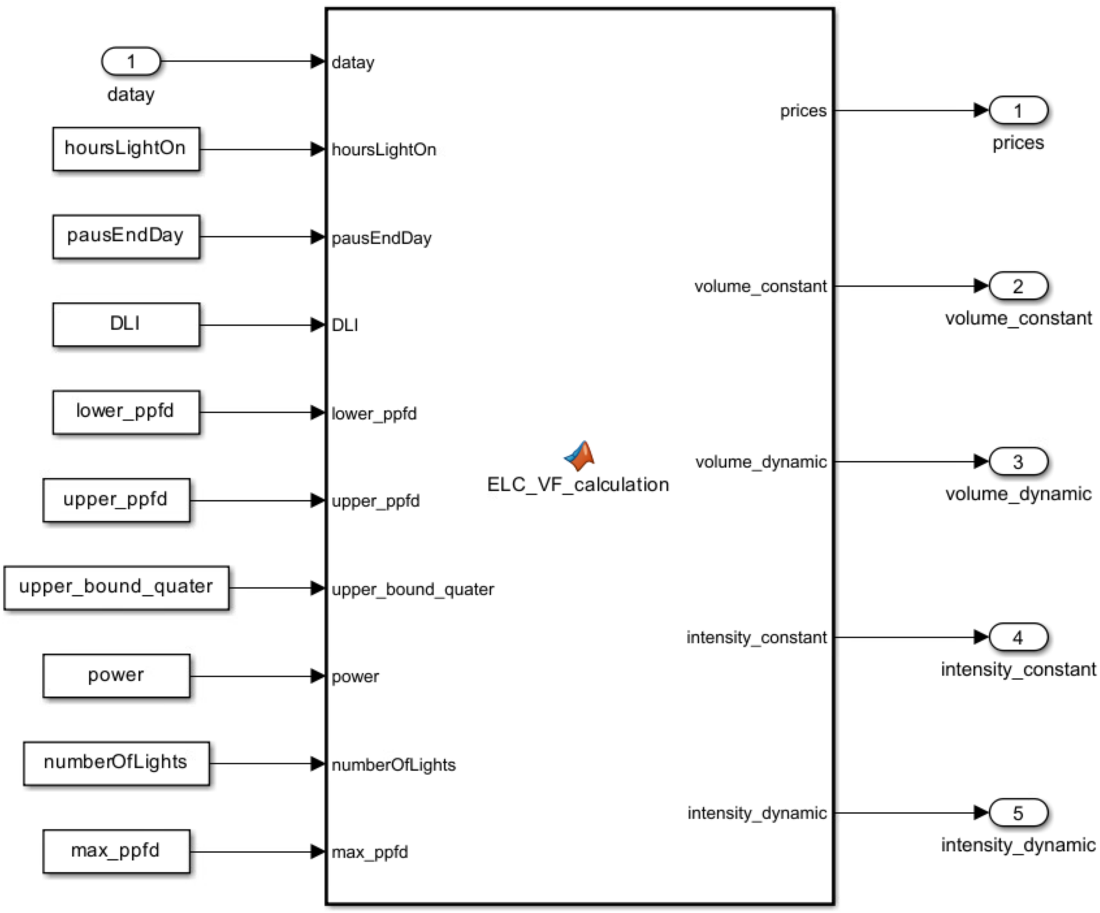

Figure 3.8: Created function in Simulink.

Figure 3.9: Data from Nord Pool from Germany 2024-03-24.

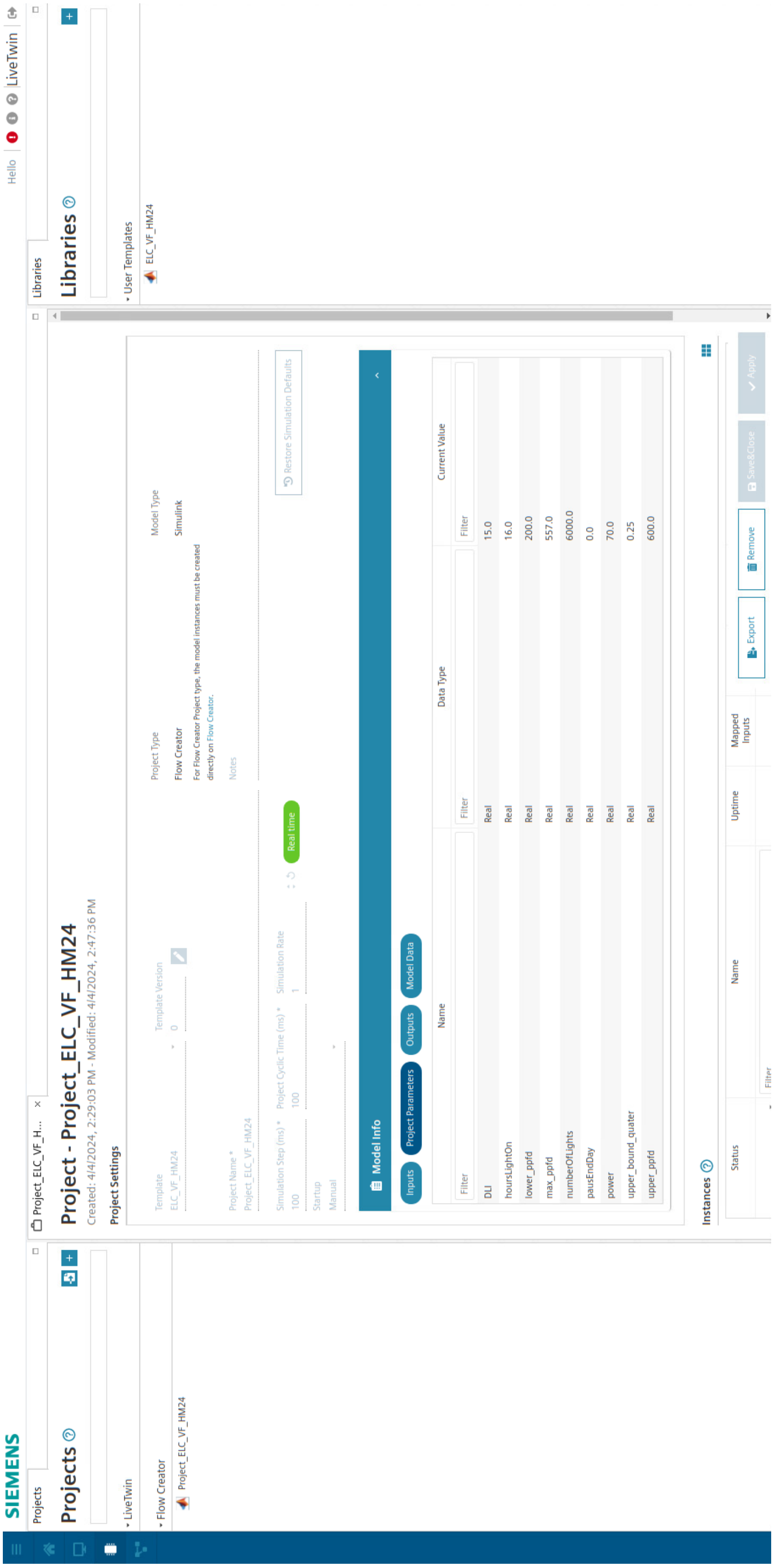

Figure 3.10: Input parameters in LiveTwin.

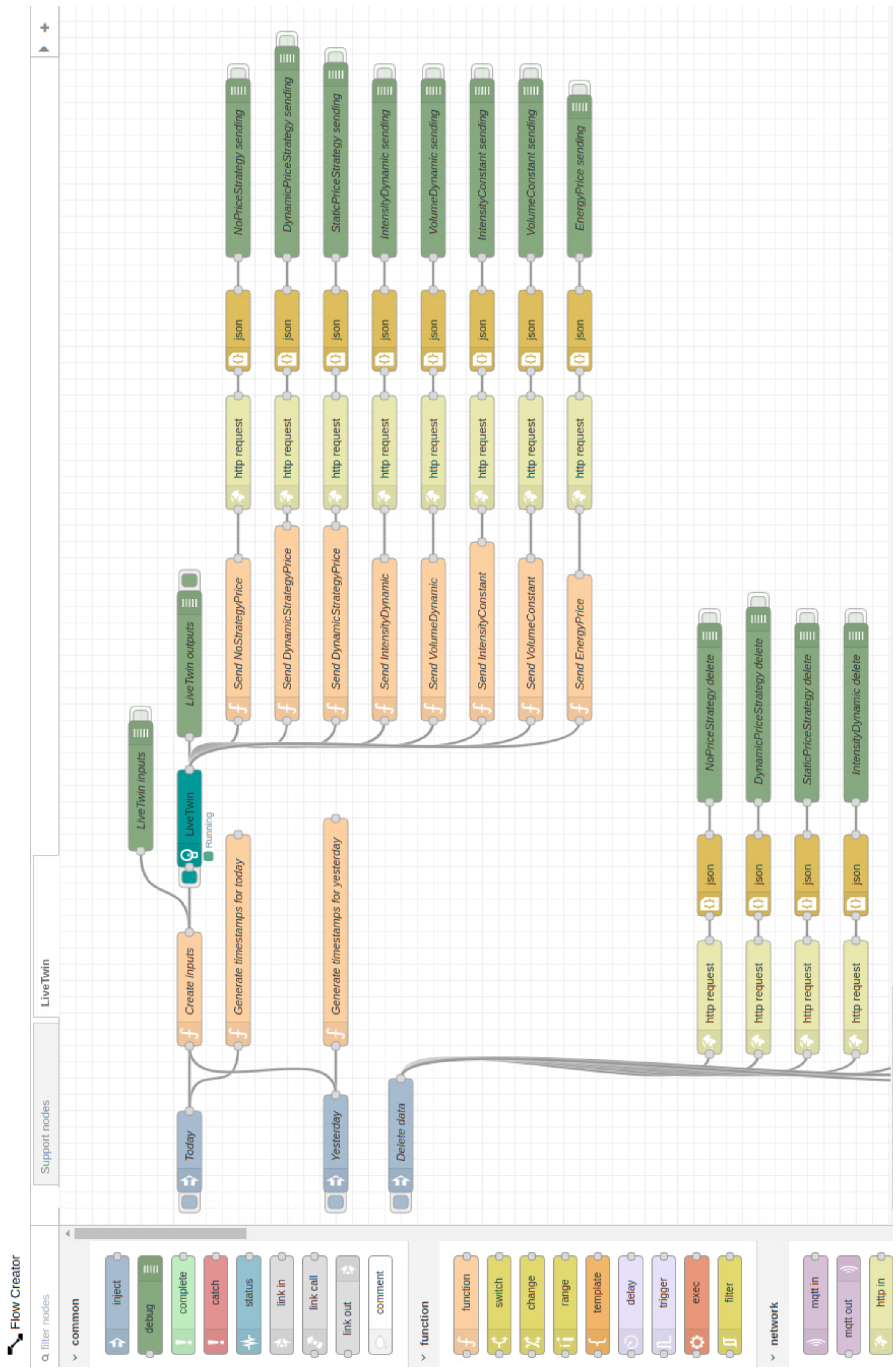

Figure 3.11: Flow Creator between LiveTwin and Energy Manager.

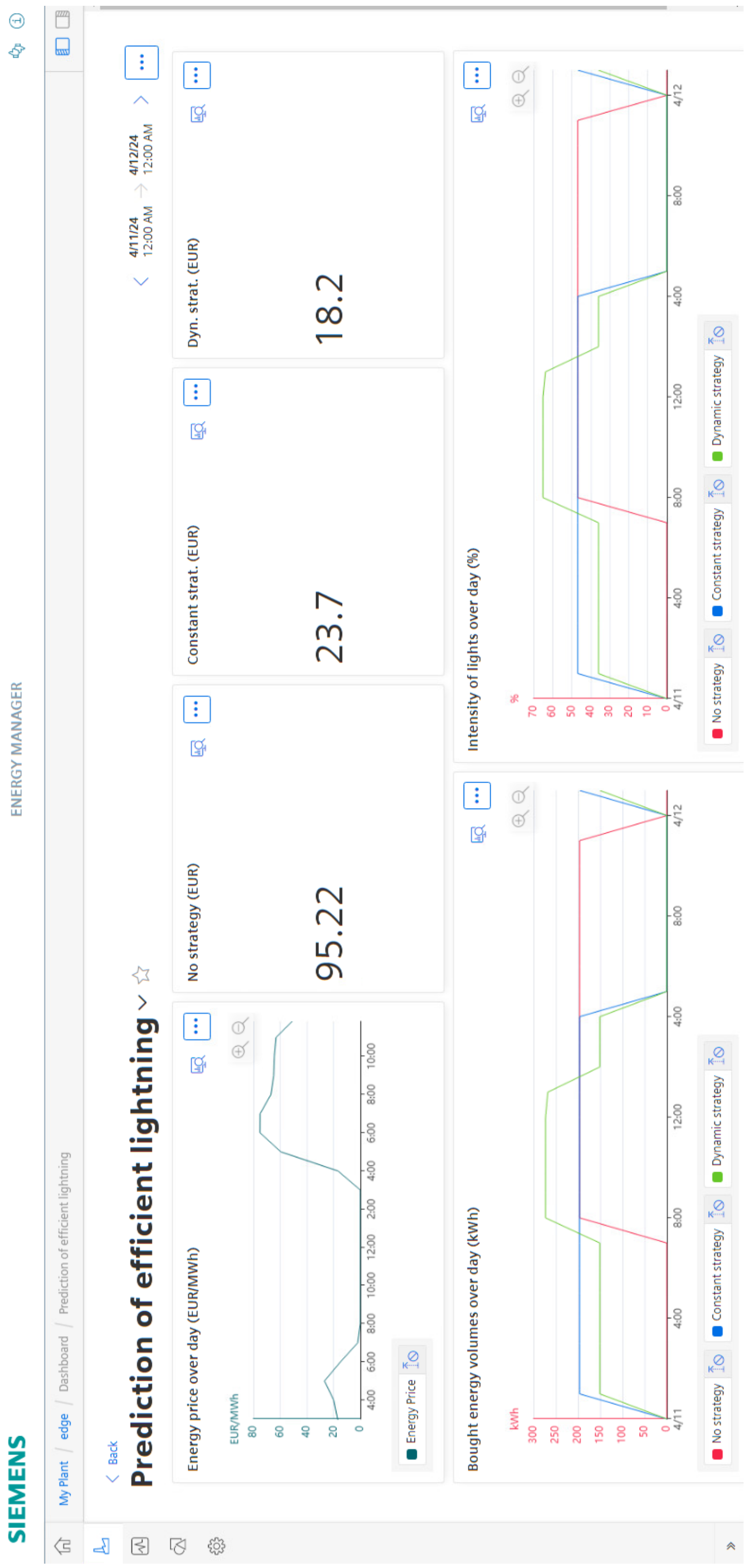

Figure 3.12: The results of lighting strategies for data from Germany on 2024-03-24.

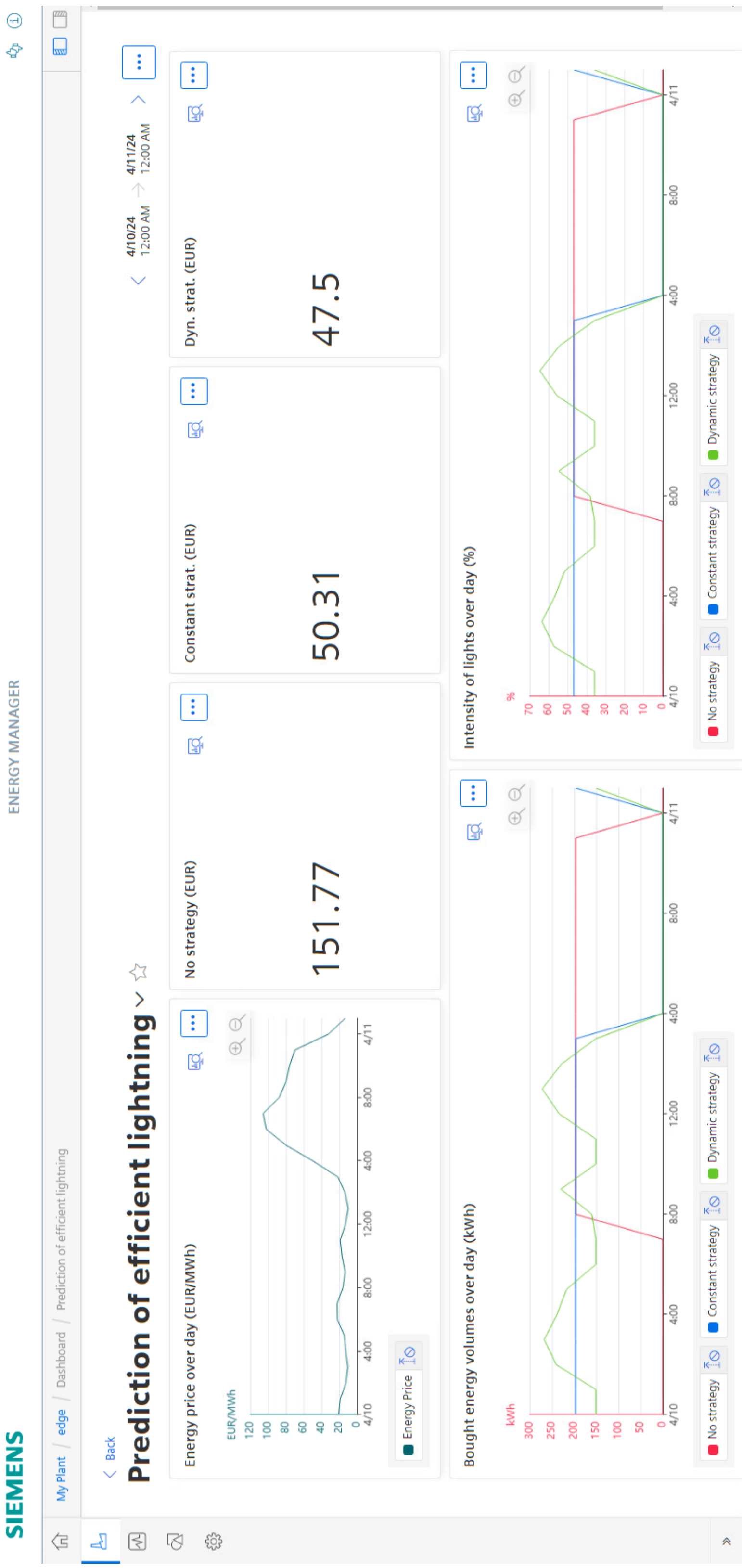

Figure 3.13: The results of lighting strategies for data from Germany on 2024-03-29.

Multi-criterial optimisation

4

This chapter starts with describing the basic concepts of multi-criterial optimisation. The optimisation problem is defined and the chosen LED light and crop parameters are introduced. The aim is to find one or more optimal solutions, which consists of combinations of a lighting strategy together with an optimal light source. First, the Pareto-optimal set of solutions is found, and then only one solution is found based on the simple additive weighted sum method (SAW). The input data are normalised by multiple techniques and weighted to evaluate the results of SAW. The chapter also covers efficiency of the solutions.

The notations of Master's thesis follows the book Multi-criteria Optimization by Ehrgott (2005) [41]. The presented definitions are from the first three chapters of the book. These chapters cover introduction to multi-criteria optimization, efficiency of solutions and the weighted sum method.

4.1 Introduction

An objective function, also known as a fitness or cost function, is a real-valued function which values are minimised or maximised subject to the constraints. Multi-objective optimisation problems involve optimising several objective functions, whose are commonly in conflict. Generally, the problem is formulated as

$$\min_{x \in \chi} (f_1(x), f_2(x), ..., f_n(x)), n \in \mathbb{N}, \tag{4.1}$$

where n is a number of objective functions and χ is a set of feasible solutions. While talking about multi-criterial optimisation, it is important to distinguish between three different spaces: the feasible set, the decision space, and the objective (or criterion) space. χ is called the feasible set, and it involves all the possible realisations of the decision space [41]. In other words, the feasible set is a subset of the decision space. Criterion or objective space contain space of images of the objective function mapping. For example, if the feasible set is

$$\chi = \{x \in \mathbb{R} : x \geq 0\},$$

and the objective function are

$$f_i(x) : \chi \rightarrow \mathbb{R}, i = \{1, 2, ..n\},$$

then the decision space is $\mathbb{R}$ and criterion space is $\mathbb{R}^n$.

Maximum, supremum or infimum are used instead of minimum. If the objective functions are maximised, then the initial problem 4.1 is rewritten to form

$$\max\ f(x) = -\min\ -f(x). \tag{4.2}$$

Proof. Let m be maximum of function A, i.e. $\exists a \in A$ such that $\forall a : a < m$. Let $-A$ denotes set $\{-a : a \in A\}$ and then apply that $-m \in -A$ and $\forall a : -m < -a$. It means that $-m$ is minimum of $-A$. Denoting $m = max(A)$ and $-m = min(-A)$ then is true that

$$max(A) = m = -(-m) = -min(-A).$$

4.2 Optimisation problem definition

In the following, all objective functions are stated, and then the problem can be solved based on different approaches. The objective functions are categorized in two types. The first one focuses on costs and the second takes into account the LED lights parameters and their influence on crop. The objective functions that involves costs are

- initial costs of lamps $f_1(x)$
- daily costs $f_2(x)$

Objective functions that involve LED lights parameters are:

- lamp wear $f_3(x)$
- IP class of water protection $f_4(x)$
- warranty $f_5(x)$
- light efficacy $f_6(x)$
- colour possibilities $f_7(x)$

The initial costs consist of the new bought equipment in the vertical farm. The first objective function aim to minimise the initial cost of the new lamps

$$f_1(x) = p(x)N_l(x),$$

where p is price of one light and N_l is a number of needed lights. The number of light is different for constant and dynamic strategies, as the dynamic strategy necessitates higher light intensity. The number of lights depends on the size of the cultivation area and amount of the layers, together denoted as A. It also depends on light specification, such as length l, beamwidth b_w, lamp photon flux PPF and also crop intensity PPFD. Formally

$$N_l(x) = N_l(A, l, b_w, \text{PPF}, \text{PPFD}).$$

Given a specific farm, lights and crop, all the parameters are fixed, except PPFD. The PPFD is influenced by the lighting strategy. For the constant strategy, the number of lights is known, since there is only one value of PPFD. For dynamic strategy, the number of lights is chosen, such that it is possible to reach PPFD_{max}.

The second objective function relate to daily costs

$$f_2(x) = c(s),$$

where $c(s)$ is price for one day based on the chosen strategy s. When considering the cost of strategies for a single day, some are less expensive. Compared strategies are constant continuous (CC), dynamic continuous (DC), constant intermittent (CI) and dynamic intermittent (DI). It applies that

$$c(\text{DC}) \leq c(\text{CC}),$$
$$c(\text{DI}) \leq c(\text{CI}).$$

In these cases, optimization is not necessary, since both dynamic strategies have lower or the same price as the constant strategies. This does not apply while comparing strategies with different amount of hours of light on or different pauses between lighting periods. Other combinations of strategies are not directly comparable. This can be demonstrated in a simple example. For this example, one day (period over is optimised) lasts 3 hours. Table 4.1 presents two possible prices of energy in two days. In the first day, the price of energy of the first hour is 1, the price of second hour is 0 and the price of the third hour in that day is again 1. Prices in the table under each strategy are for the whole day. The light is on for 2 hours in each strategy, and the minimal amount of consumed energy is equal to 0.002 MWh. In intermittent strategy, the light is on the first and the third hour. In continuous strategy, the light can be switched on the first and the second hour or the second

Table 4.1: Prices of strategies for two days with 3-hour long day period. 3 prices [1,0,1] and [0,1,0] are prices of energy for each hour.

prices	DC	DI	CC	CI
[1,0,1]	0.002	2	1	2
[0,1,0]	0.002	0	1	0

and the third hour. In table can be seen that DI could be higher or lower than CC. The same applies for combinations (DC, CI), (CC, CI), (DC, DI).

The daily costs of lighting $f_2(x)$ depends on the strategy, chosen LED light and the prices of energy during a day. The daily costs are calculated as an average prices over some specified time.

The next costs for vertical farms are associated with the parameters of the light sources. The first one is lamp wear ($f_3(x)$). The aim is to find the optimal time after which is necessary to change the light because it does not fulfil crop requirements for needed amount of intensity PPFD. The optimisation problems try to find the lights that do not have to be changed often. Formally

$$f_3(x) = T,$$

where

$$T = \inf\{t \geq 0 : \phi(t) \leq \mathrm{PPFD}_{\max}\},$$

T is the first time, when the intensity $\phi(t)$ in time t is equal or lower than maximal possible amount of intensity given for crop $\mathrm{PPFD}_{\max}$.

IP class of water protection I and warranty w are two parameters, that are also taken into account while choosing the optimal LED light. The objectives are following

$$f_4(x) = I \text{ and } f_5(x) = w.$$

Another parameter important for lights is their efficiency to convert energy into light. That is called light efficacy $E \in \mathbb{R}^+$. It is a ratio

$$E = \frac{\mathrm{PPF}}{P_n},$$

where PPF is the photosynthetic photon flux measured over a specific spectrum (mostly in the visible, 300-780nm) and P_n is a nominal power of light. Higher efficacy means that the light emits more moles per unit of nominal power P_n. It is measured in $\frac{\mu\mathrm{mol}}{\mathrm{J}}$. The efficacy influences the overall energy costs. Daily costs of light, unlike efficacy, are also influenced by the number of lights and have more complex computation. The objective function is

$$f_6(x) = E.$$

The next parameter to consider is the colour possibilities of LED lamps. Different crops require specific light colours. The objective function is stated as

$$f_7(x) = o,$$

where $o \in \mathbb{R}$ represents the colours. If all colour that the crop need are involved in colour options of lamps, then the objective function is assigned with high number, e.g., $o = 100$. If the LED lamp does not have all needed colours, however, contains white colour, then it is assigned with lower number, e.g., $o = 50$. White composed of all colours, however, the needed colours are not prominent. If light does not include all needed colours, then o is set to the matching number of colours between light and crop.

All above mentioned objective functions can be summarized to one problem. The aim is to find

$$\begin{gathered} \min\ (f_1(x), f_2(x), -f_3(x), -f_4(x), -f_5(x), -f_6(x), -f_7(x)) \\ \text{subject to } x \in \chi, \end{gathered} \tag{4.3}$$

where χ is a set of feasible solutions. The feasible solutions consist of possible LED lights and their parameters. Based on 4.2 the second part of objective function should be in form $-min\ (-f_3(x), -f_4(x), -f_5(x), -f_6(x), -f_7(x))$. However, the definition 4.3 does not influence the solution in the following Section 4.4. The minimisation 4.3 consist of objective functions from $\mathbb{R}$ and $\mathbb{N}_0$. The problem belongs to mixed integer multiobjective optimisation.

4.3 Chosen LED lights and crop parameters

Romaine lettuce and strawberries are chosen for comparison of the results of multi-criterial problem 4.3. These two were chosen because of their different light needs. Lettuce need a lower intensity of light for a long time. On the other hand, strawberries needs FR light for fruiting and higher intensity of light for shorter time. Their input parameters are shown in Table 4.2.

Table 4.3 shows all the specifications of LED Arize Life2 light that are used for multi-criterial optimisation [43, 44]. The prices are in dollars and have to be exchanged to euros. Exchange rate of the European Central Bank is used for the conversion on 5.4.2024. The exchange rate for 1 USD is 0.9224 EUR. The thesis attachment includes screenshots of LED light prices, the exchange rate and PDFs of light manuals. The Table 4.4 includes information about lights from companies

Table 4.2: The input parameters for romaine lettuce and for strawberries [42].

	romaine lettuce	strawberry
hours light on	16	12
intermittent strategy	4x4 with 1 hour pauses	4x3 with two hours pauses
DLI	12	15
minimal PPFD	150	300
maximal PPFD	300	600
colour	R,B,G	R,B,FR

Lumatek, Secret Jardin and Philips. All the prices in Table are in CZK, and they were exchanged by the rate 1 CZK to 0.03955 EUR. In the tables, some values are red. These values are not stated by the light producer. Warranty is set for 2 years, since it is stated by EU law that the consumer must be given a minimum of 2-year guarantee as a protection against faulty goods. Not all light parameters are stated on the market. Missing information is mostly beam width, warranty or instead of PPF are stated lumens. For the Philips Gen 1 the beam width is not stated. Since all the other generation have the beam width 120 degrees, assumption can be made that the first generation has the same beam span. Large producers of lamps state all of the above except price. For the Philips and the CoolGrow lights the prices are chosen. These prices do not have to correspond to reality and are estimated just for the comparison of multi-criterial techniques. There was just one rule while assigning price to each lamp and that is that lamp with more colour options, length and PPF has higher or the same price as the lamps with lower parameters from the same producer.

For these input parameters, realisations of each objective function are computed. All of these realisations create a decision matrix **D** with alternatives in rows and criteria in columns. The element of the decision matrix d_{ij} is the i-th alternative with respect to the j-th criterion. Further, m denotes the number of alternatives and n is the number of criteria or objective functions.

Table 4.3: Arize Life2 lights with beam span 126 degrees, degradation L90B50(54 000), IP66 and warranty 5 years.

	A1	A2	A3	A4	A5	A6
power [W]	32	32	30	32	64	64
PPF [$\frac{\mu mol}{s}$]	103	89	90	90	206	180
length [m]	1.22	1.22	1.22	1.22	2.44	2.44
price [EUR]	115.3	115.3	115.3	115.3	212.2	212.2
colour	B,R	B,R,G,FR	B,R,FR	B,R,G	B,R	B,R,G

Table 4.4: Lights from various producers. All the light has intensity L90B0.

	Lumatek	S. J. 40W	S. J. Kit	Philips Gen1
power [W]	100	40	80	43
PPF [$\frac{\mu mol}{s}$]	295	114	160	62.5
length [m]	1.148	0.7	0.905	1.53
beam span [°]	120	120	120	120
price [EUR]	197.3	66.0	152.2	148.1
colour	W+R	W+R	W	R+B+FR
degradation [hours]	60000	30000	30000	25000
IP	65	65	65	66
warranty [years]	5	2	2	2

Table 4.5: Philips lights with L95(36000), IP66 and warranty 3 years. All parameters correspond to reality except the prices marked with red colour.

	P1	P2	P3	P4	P5
power [W]	70	88	70	88	88
PPF [$\frac{\mu mol}{s}$]	168	210	168	210	210
length [m]	120	150	120	150	240
beam span [°]	120	120	120	120	140
price [EUR]	180	200	190	220	300
colour	R,B	R,B	R,B,FR	R,B,FR	R,B,FR

Table 4.6: CoolGrow lights with length 1.16 m, L95B50(50000),IP66, warranty 5 years.

	C6	C7	C8	C9	C10	C11	C12	C13
power [W]	60	60	60	60	100	100	100	100
PPF [$\frac{\mu mol}{s}$]	210	210	210	210	350	350	350	350
beam span [°]	120	150	120	150	120	150	120	150
price [EUR]	200	200	200	200	250	250	250	250
colour	R,B,W	R,B,W	R,B,FR	R,B,FR	R,B,W	R,B,W	R,B,FR	R,B,FR

4.4 Optimality of solutions

Multi-objective optimisation (MOO) focuses on finding the set of optimal solutions. For solutions in this set, improving one objective comes at the expense of worsening another. This set of solutions is called Pareto-optimal and the image of feasible set is called nondominated set. Pareto-optimal and nondominated sets are described in Definition 4.4.1 and Definition 4.4.2 [41, 45].

Definition 4.4.1 (Pareto-optimal) *A feasible solution $\hat{x} \in \chi$ is called efficient or Pareto-optimal, if there is no other $x \in \chi$ such that*

$$
\begin{aligned}
f_i(x) &\leq f_i(\hat{x}) \quad \text{for all } i \in \{1, 2, ..., n\} \\
f_j(x) &< f_j(\hat{x}) \quad \text{for at least one } j \in \{1, 2, ..., n\}
\end{aligned}
$$

Definition 4.4.2 (nondominated set) *If $\hat{x}$ is efficient, $f(\hat{x})$ is called nondominated point. If x_1,x_2 $\in \chi$ and $f(x_1) \leq f(x_2)$ it is said that x_1 dominates x_2 and $f(x_1)$ dominates $f(x_2)$. The set of all efficient solutions $\hat{x} \in \chi$ is denoted χ_E. The set of all nondominated points $\hat{y} = f(\hat{x})$, where $\hat{x} \in \chi_E$ is denoted γ_N and called the nondominated set.*

In addition to the Pareto optimal solution, there are also weakly and strictly Pareto optimal solutions, as defined in Definition 4.4.3 and Definition 4.4.4. The difference between them is in the componentwise comparisons of the objective functions.

Definition 4.4.3 (weakly Pareto-optimal) *A feasible solution $\hat{x} \in \chi$ is called weakly Pareto-optimal if there is no $x \in \chi$ such that $f(x) < f(\hat{x})$, i.e. $f_k(x) < f_k(\hat{x})$ for all $k \in \{1, ..., n\}$. The point $\hat{y} = f(\hat{x})$ is then called weakly nondominated. The weakly efficient and nondominated set are denoted χ_{wE} and γ_{wN}.*

When χ takes on many realisations or is continuous, the weakly Pareto-optimal set is the approximation of efficient solution.

Definition 4.4.4 (strictly Pareto-optimal) *A feasible solution $\hat{x} \in \chi$ is called strictly Pareto-optimal if there is no $x \in \chi$ such that $f(x) \leqq f(\hat{x})$. The point $\hat{y} = f(\hat{x})$ is then called strictly nondominated. The strictly efficient and nondominated set are denoted χ_{sE} and γ_{sN}.*

The strictly Pareto-optimal set of solution is not suitable to search, because objective function with identical values to another would not be part of the solution. Different LED lamps with the same set of parameters are equally suitable for IVF and there is no reason why one should not be in the optimal set. From the definitions above, it is obvious that

$$\chi_{sE} \subset \chi_E \subset \chi_{wE}.$$

The thesis introduces a brute force algorithm and weighted sum method. The Pareto-optimal solutions in 4.4.1 are also called proper efficient solutions, to distinguished them from weakly and strictly Pareto-optimal solutions. The optimality is more commented in each section dedicated to optimising algorithm.

4.5 Brute force algorithm

The brute force algorithm aim to find the solution by componentwise comparing of objective functions. The focus is on finding the Pareto-optimal set of solutions χ_E. The focus is not on weakly and strictly Pareto-optimal, because the weakly Pareto-optimal set is unnecessarily larger and the strictly Pareto-optimal set does not involve identical results for different LED lights and strategies. The algorithm compares rows of the decision matrix **D**. If all values in row k of **D** are equal and at least one value is higher than in row l, then the row k is not part of the nondominated set. It is formulated as $\exists k, l \in i; k \neq l$:

$$
\begin{aligned}
&f_j(x^k) \geq f_j(x^l) && \forall j \in \{1, 2, ..., n\} \\
&f_j(x^k) > f_j(x^l) && \text{for at least one } j \in \{1, 2, ..., n\}
\end{aligned}
$$

where j is j-th objective function, and $i \in \{1, 2, ..., m\}$ is set of rows. Then

$$k \notin \chi_E.$$

In the set of solutions does not exist two LED lamps and strategies with the same values for each objective function. It means that the founded solution is also strictly Pareto-optimal χ_{sE}. The result can be seen in the attached Matlab file `bruteforce.m`.

Brute force is inappropriate for large number of rows in **D**, because of rapid increase of computational complexity. From all the m number of alternatives (rows), two possible alternatives are compared in each step. That is variation of 2 elements from m. Which is equal to $\frac{m!}{(m-2)!}$. In other words, the first row is compared with remaining rows. Which is $m - 1$ operations. The same is done for second, third, ..., last row. This means that in the worst case, $m(m - 1)$ operations need to be done. That is true for objective function with one criterion. For n criteria, each one needs to be checked, which makes for $nm(m - 1)$ operations. Thus, the computation complexity is $O(nm^2)$.

In Figure 4.1, Pareto-optimal lights for strawberries can be seen. The picture shows only the dependency of the objective function $f_2(x)$, daily costs of light energy, on the objective function $f_1(x)$, initial costs of lights. The reason why some dominated point in Figure 4.1 looks Pareto-optimal is, that only the dependency of two objective functions is depicted. The light can be equal with objective function with $f_1(x)$ and $f_2(x)$, however, worse in others.

The Pareto-optimal lights and their strategies are shown for strawberries in Table 4.8 and for romaine lettuce are in Table 4.7. Secret Jardin kit, Philips Gen1 and Philips light P5 are not used for strawberries, because the light's PPF is too low for light length and beamwidth. They can be used for lettuce since lettuce need lower amount of light intensity PPFD.

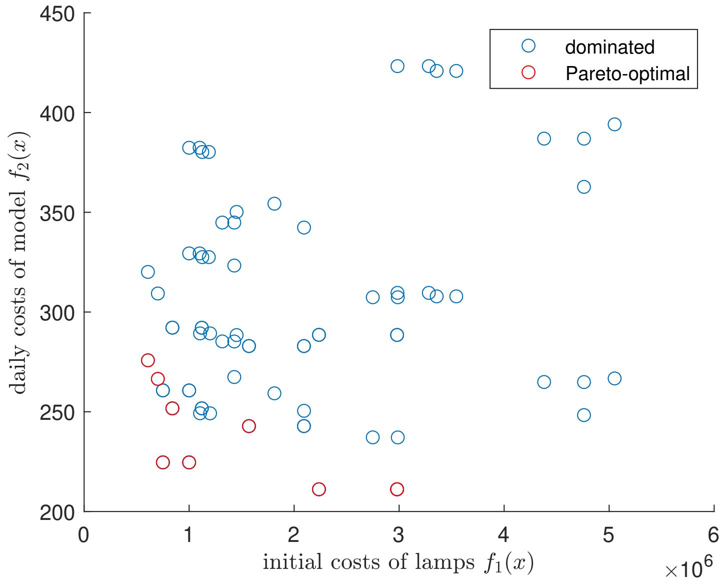

Figure 4.1: Pareto-optimal set for strawberries is depicted by red circles. Blue circles belong to dominated lights.

In both cases, only the constant intermittent and dynamic intermittent strategies are in the Pareto-optimal set. Because romaine lettuce need lower intensity, the maximum of values of initial and averages daily cost are lower. The light for chosen crops differ mostly because of colour specification of crop.

Table 4.7: Pareto-optimal lights for romaine lettuce.

light	strat.	initial c.	daily c.	lamp wear	IP	warr.	efficacy	col.
A6	CI	758 190	248.08	56.18	6	5	2.81	100
Lumatek	CI	421 432	231.76	56.20	5	5	2.95	50
S. J. 40W	CI	364 782	239.91	28.08	5	2	2.85	50
S. J. Kit	CI	600 200	195.37	96.12	6	5	3.50	50
C6	CI	672 600	218.92	169.02	6	5	3.50	50
C10	CI	450 250	195.37	96.14	6	5	3.50	50
C11	CI	504 500	218.94	169.20	6	5	3.50	50
A4	DI	1 397 666	232.32	56.18	6	5	2.81	100
A6	DI	1 286 144	232.33	56.18	6	5	2.81	100
C6	DI	1 079 400	183.70	96.12	6	5	3.50	50
C7	DI	1 008 400	201.89	169.02	6	5	3.50	50
C10	DI	809 500	183.71	96.14	6	5	3.50	50
C11	DI	756 250	201.89	169.20	6	5	3.50	50

Table 4.8: Pareto-optimal lights for strawberries.

light	strat.	initial c.	daily c.	lamp wear	IP	warr.	efficacy	col.
Lumatek	CI	702 388	266.47	55.00	5	5	2.95	50
S. J. 40W	CI	607 926	275.80	27.50	5	2	2.85	50
C8	CI	1 000 200	224.62	94.17	6	5	3.50	100
C12	CI	750 250	224.62	94.15	6	5	3.50	100
C13	CI	840 750	251.71	167.05	6	5	3.50	100
C8	DI	2 980 600	211.15	94.17	6	5	3.50	100
C12	DI	2 236 500	211.15	94.15	6	5	3.50	100
C13	DI	1 570 000	242.80	167.05	6	5	3.50	100

4.5.1 Pareto-optimal fronts

The set of lights that is Pareto-optimal is also known as the Pareto-optimal front. Removing the Pareto-optimal front from the initial set enables to find the second Pareto-optimal front. This new front is optimal relative to the remaining set of lights. Continuing this technique, the lights are divided into Pareto-optimal fronts.

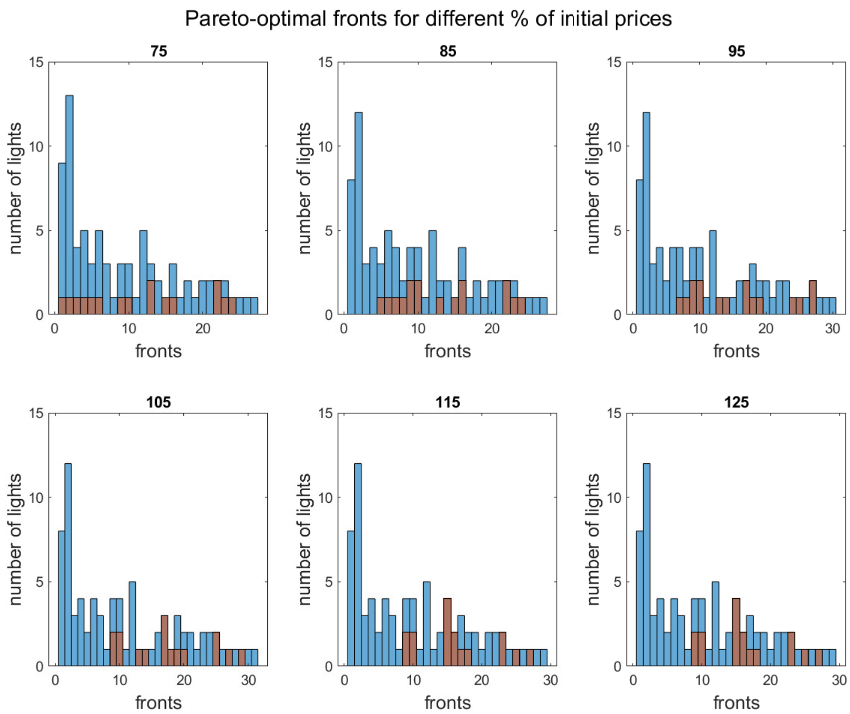

Figure 4.2: Histogram of number of Philips LED light in each Pareto-optimal fronts for strawberries.

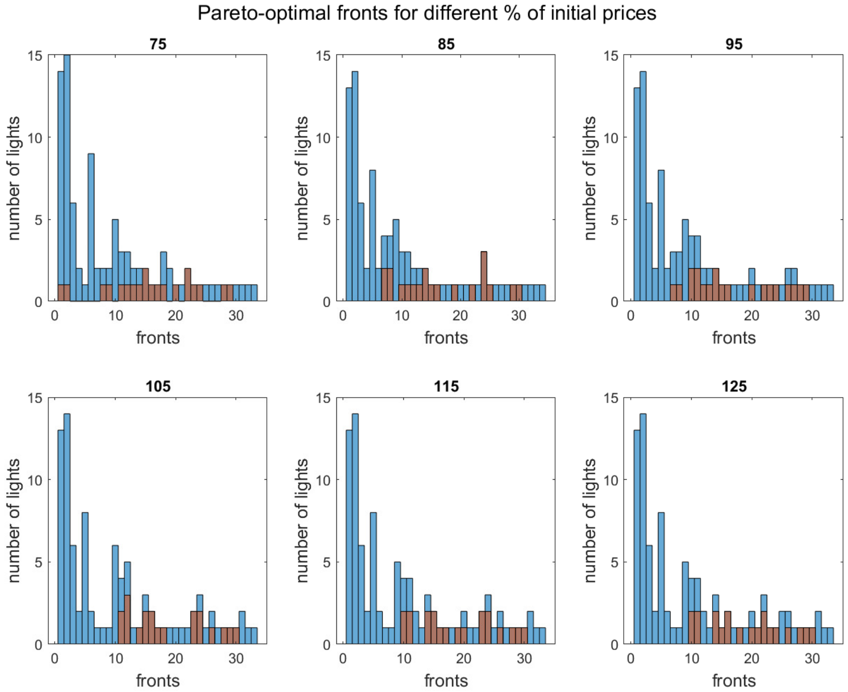

Figure 4.3: Histogram of number of Philips LED light in each Pareto-optimal fronts for romaine lettuce.

This work shows an example of Pareto-optimal sets for different prices of Philips lights. The price of light influences only the first criterion, which is the initial costs. In Figures 4.2 and 4.3 can be seen histograms of Pareto-optimal lights in each front. The first picture shows histogram for strawberries and the second shows histogram for romaine lettuce. The histograms are for different percentage of the initial price of LED lights.

4.6 The Weighted Sum Method

The Brute force found a set of optimal solutions. The focus is now to find only one and the best solution based on various criteria. The chosen alternative is not necessarily the Pareto-optimal solution. The well known and commonly used method in multi-criteria decision-making (MCDM) is simple additive weighted sum method (SAW). The weighted sum method is based on solving a single objective function. The weight is assigned to each criterion, and the weighted criteria are summed. The inputs data needs to be normalised, so the weights have corresponding importance.

The problem is formulated as

$$\max_{x \in \chi} \sum_{k=1}^{n} w_k f_k(x), \tag{4.4}$$

$$w_k \in \langle 0, 1 \rangle \text{ and } \sum_{k=1}^{n} w_k = 1,$$

$$f_k(x) \in \langle 0, 1 \rangle.$$

where w_k is the weight of kth objective function $k \in \{1, 2...n\}$ and $n \in N$ is a number of criteria or results of objective functions $f_1(x), f_2(x)...f_n(x)$.

4.6.1 Optimality of SAW

This section is dedicated to determine if the solutions are strictly, properly or weakly efficient. The topic is covered in Chapter 3 in Ehrgott [41] together with proofs. The sign of the weights and the convexity of the feasible space play essential roles in determining the optimality of solutions. The Proposition 1 and Proposition 2 summarizes the results. $\mathbb{R}^n_{\geq}$ is in the propositions the n-dimensional real space where each component of the vector is non-negative.

Proposition 1 *Suppose that $\hat{x}$ is an optimal solution of the weighted sum optimisation problem 4.4 and $w = (w_1, w_2, ..., w_n)$ is a vector of weights. Then the following statements hold.*

1. *If $w \in \mathbb{R}^n_{\geq}$ then $\hat{x} \in \chi_{wE}$*
2. *If $w \in \mathbb{R}^n_{>}$ then $\hat{x} \in \chi_E$*

The proposition 1 states that with assumption that the solution is optimal, non-negative weights guarantee weakly efficiency, and positive weights guarantee proper efficiency. How to ensure that the solution is optimal? The Proposition 2 answer that question. Before the Proposition 2, set of all solutions, Minkowski sum and the convexity of a set needs to be defined. The set of all solutions $S(w, \gamma)$ for fixed $w \in \mathbb{R}^n_{\geq}$ is

$$S(w, \gamma) = \{\hat{y} \in \gamma : \langle w, \hat{y} \rangle = \min_{y \in \gamma} \langle w, y \rangle\},$$

where γ is a set of optimal points with respect to w. Second, is the definition of convexity. For the definition of convexity, Minkowski sum needs to be defined.

Definition 4.6.1 (Minkowski sum) *Let $S_1, S_2 \subset \mathbb{R}^n$ be two sets. The Minkowski sum of S_1 and S_2 is $S_1 \oplus S_2 := \{s_1 + s_2 : s_1 \in S_1, s_2 \in S_2\}$.*

The Minkowski sum is the sum of all elements, e.g., $S_1 = \{1, 2\}, S_2 = \{-3, 0\}$ then $S_1 \oplus S_2 = \{-2, -1, 1, 2\}$. Then, the convexity is defined.

Definition 4.6.2 (Convexity of set) *A set $s \in \mathbb{R}^n$ is called $\mathbb{R}^n_{\geq}$-convex, if $s \oplus \mathbb{R}^n_{\geq}$ is convex.*

Finally, the Proposition 2 is stated.

Proposition 2 *If γ is $\mathbb{R}^n_{\geq}$-convex, then $S(\gamma) = \gamma_{wN}$,*

Some objective functions involve real values while others include integer variables, making the problem non-convex due to the presence of the discrete variables. Consequently, the solutions found are not guaranteed to be optimal. However, optimal solutions have already been identified using a brute force algorithm. The weighted sum method aims to find the most suitable one. Next, the chapter introduces the individual steps in SAW:

1. Determination of the normalisation technique for the objective functions and determination of the cost/benefit criteria

2. Assignment of weights to each objective function

3. Evaluation of solutions

4.6.2 Determination of the normalisation technique

Firstly, objective functions are divided between costs and benefit classes. Costs criteria involve the initial cost of lamps and the daily costs. These criteria are minimised. The rest of the criteria are maximised. The benefit or maximised criteria are lamp wear, IP class, warranty, light efficacy and colour possibilities.

The values of objective functions have to be normalised before applying any method, such that the weights have the same relative importance. Different normalisation techniques can have various results, and some could be more suitable than others. Using previous research [46, 47, 48, 49, 50], the criteria for cost and benefit normalisation are chosen, see Table 4.9.

The normalisation techniques should be compared by more tests to ensure the chosen method is appropriate for the specific context. The normalisation techniques are compared based on order of alternatives. The order can be compared by Spearman's Rank Correlation coefficient, Kendall τ and rank consistency index (RCI). The robustness is checked by standard deviation. The coefficients are now defined [51].

Table 4.9: Table shows different normalisation techniques for benefit and cost criteria. r_{ij} is an element of matrix R of all possible values for different type of light and lighting strategy. Each column i represents all realisation of decision space of one objective function f_i, e.g. $(f_1(x_1), f_1(x_2), ...)$. Each row j represent one output from each objective function $f_1(x_i), f_2(x_i), ...$

normalisation technique	benefit criteria	cost criteria
linear max (N1)	$\frac{r_{ij}}{max(r_i)}$	$1 - \frac{r_{ij}}{max(r_i)}$
linear max-min (N2)	$\frac{r_{ij}-min(r_i)}{max(r_i)-min(r_i)}$	$\frac{max(r_i)-r_{ij}}{max(r_i)-min(r_i)}$
linear sum (N3)	$\frac{r_{ij}}{\sum_{i=1}^{m} r_i}$	$\frac{\frac{1}{r_{ij}}}{\frac{1}{\sum_{i=1}^{m} r_i}}$
vector min (N4)	$\frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_i^2}}$	$1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_i^2}}$
logarithmic (N5)	$\frac{ln(r_{ij})}{ln(\prod_{i=1}^{m} r_i)}$	$\frac{1-\frac{ln(r_{ij})}{ln(\prod_{i=1}^{m} r_i)}}{m-1}$

Definition 4.6.3 (Spearman's rank correlation coefficient) *Let* $(\mathbf{X}, \mathbf{Y})$ *be two sets of independent and identically distributed observations, where* $R_1, R_2, ..., R_m$ *is order of* $x_1, x_2, ..., x_m$ *and* $Q_1, Q_2, ..., Q_m$ *is order of* $y_1, y_2, ..., y_m$. *Then the Spearman's Rank Correlation for distinct integers is*

$$\rho_s = 1 - \frac{6\sum_i (R_i - Q_i)^2}{m(m^2 - 1)},$$

where $i \in \{1, 2, ..., m\}$ *and* m *is the number of alternatives.*

If ρ_s is closer to 1, then it has more similar order than lower values of ρ_s. If two or more observation of $\mathbf{X}$ or $\mathbf{Y}$ are equal, then they are assigned the mean value of their respective rank.

Definition 4.6.4 (Kendall's tau) (x_i, y_i) *and* (x_j, y_j) *are two realisation of the sets* $(\mathbf{X}, \mathbf{Y})$. *The realisation are concordant if* $(x_i - x_j)(y_i - y_j) > 0$. *The realisations are disconcordant if* $(x_i - x_j)(y_i - y_j) < 0$. *The Kendall* τ *is*

$$\tau = \frac{c - d}{\binom{m}{2}},$$

where c *is number of concordant,* d *is number of disconcordant and* $\binom{m}{2} = \frac{m(m-1)}{2}$ *is a number of all pairs.*

Kendall's tau, the same as Spearman's rank correlation coefficient, is value between $\langle -1, 1\rangle$. The higher the value is, the more identical their ranking between the two variables is. Low value indicates dissimilar ranking.

The RCI measures the total number of similarities between one normalization technique and all others. The RCI is calculated with a consistency weight (CW), which represents the percentage of similar normalization techniques. The CW is equal to

$$\mathrm{CW} = \frac{p}{n-1},$$

where n is a number of normalisation techniques and p is a number of normalisation techniques with which the ranks are compared. Example of calculation of RCI for the first normalisation technique out of 4 possible is computed as

$$\begin{aligned}\mathrm{RCI}_1 =&(T_{1234}(CW=1)+T_{123}(CW=\frac{2}{3})+T_{124}(CW=\frac{2}{3})+T_{134}(CW=\frac{2}{3})\\ &+T_{12}(CW=\frac{1}{3})+T_{13}(CW=\frac{1}{3})+T_{14}(CW=\frac{1}{3}))/\mathrm{TS},\end{aligned} \tag{4.5}$$

where TS is a total number of simulations and T with lower index is a total number of times that the first normalisation technique is similar to other normalisation techniques in lower index of T.

The robustness of the normalisation techniques is measured by the standard deviation (STD). The STD measures the spread of the data from its mean. The standard deviation is estimated by sample deviation for each technique as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m}(x_i-\mu)^2}{m-1}}),$$

where x_i the is result after weighting for one alternative and one normalisation technique, μ is the mean of x_i, $i \in$ and m is a number of alternatives. The robustness of the normalisation techniques can also be checked by Minkowski distance.

The ranks are from 1 to 5. The higher is the rank, the better is the normalisation technique. For all above defined coefficients apply, that the higher they are, the higher rank is it assigned.

4.6.3 Assignment of weights to each objective function

The weight can be assigned by an expert in the field or by experimenting with mathematical models. Using previous research [48, 52], the weight criteria are chosen.

Equal Weight Method

The first method is the equal weight method, where weight come from the uniform distribution. The weights are assigned equally,

$$w_j^{EW} = \frac{1}{n},$$

where $j \in \{1, 2, ..., n\}$ is the index of weight corresponding to the j-th objective function.

Entropy Method

Another method to assign weights is the entropy method. From the decision matrix **D** are normalised values for each objective function as $P_{ij} = \frac{d_{ij}}{\sum_j d_{ij}}$. The entropy of each column is

$$e_j = -\frac{1}{ln(m)} \sum_j P_{ij} ln(P_{ij}).$$

Entropy of a random variable represents uncertainty of the variable's possible outcome. If $e_j = 0$, that means that the outcome is known. $e_j = 1$ means the highest entropy and therefore e_j has the highest uncertainty of outcome. The weight is higher for lower uncertainty

$$w_j^{EM} = \frac{1 - e_j}{\sum_j 1 - e_j}.$$

Reciprocal weights

Another approach of assigning weight is based on importance of each criterion or objective function. The objective functions with the highest importance have assigned rank $r_j = 1$, the second most important has rank $r_j = 2$ and so on until $n = 7$. There are described 2 types of rank ordering. The first rank ordering criteria is called reciprocal, where the weight of each criterion uses the reciprocal value of the ranks normalised by the sum of the reciprocals. The formula for reciprocal weights (RR) is following

$$w_j^{\mathrm{RR}} = \frac{1/r_j}{\sum_{k=1}^{n} 1/r_k}.$$

Rank Sum weight method

In this method, the ranks are subtracted from the number of criteria. The weights are normalised by dividing by the sum of the ranks. The formula for calculation of ranks

$$w_j^{\mathrm{RS}} = \frac{n - r_j + 1}{\sum_{k=1}^{n}(n - r_k + 1)}$$

can be simplified to $\frac{2(n-r_j+1)}{n(n+1)}$. The sum is divided to $\sum_{k=1}^{n}(n+1) - \sum_{k=1}^{n} r_k$, which equals $n(n+1) - \frac{1}{2}n(n+1) = \frac{1}{2}n(n+1)$. The second sum $\sum_{k=1}^{n} r_k = \frac{1}{2}n(n+1)$ can be proof for natural numbers $(r_1, r_2, ...r_n) = (1, 2, 3, ...)$ by induction.
For $n = 1$ applies that

$$\sum_{k=1}^{1} r_j = \frac{1}{2}2 = 1.$$

Induction hypotheses assume that for any n is true that

$$\sum_{k=1}^{n} r_k = \frac{1}{2}n(n+1).$$

For $n+1$ should apply $\sum_{k=1}^{n+1} r_k = \frac{1}{2}(n+1)(n+2)$. For induction step, $n+1$, apply that

$$\sum_{k=1}^{n+1} r_k = \sum_{k=1}^{n} r_k + n + 1 = \frac{1}{2}n(n+1) + n + 1 = \frac{1}{2}(n+1)(n+2).$$

4.6.4 Evaluation of normalisation techniques

Normalisation techniques are investigated for each method of assigning weights. In this section, results for all possible lights and strategies of romaine lettuce weighted by entropy method are shown. Full result can be seen in `rank_nondominated.m`". Spearman's and Kendall's coefficients are in Table 4.10 and 4.11. The ranking is different only for the two lowest means. In both cases, the best three normalisation techniques based on the coefficients are N4 (vector min), N1 (linear max) and N3 (linear sum).

Table 4.10: Spearman's rank correlation coefficients with means and rank of each normalisation technique.

	N1	N2	N3	N4	N5	mean	rank
N1		0.979	0.940	0.988	0.907	0.954	4
N2	0.979		0.871	0.945	0.839	0.909	1
N3	0.940	0.872		0.973	0.962	0.937	3
N4	0.988	0.945	0.974		0.931	0.959	5
N5	0.907	0.839	0.962	0.931		0.910	2

Table 4.11: Kendall's taus with means and rank of each normalisation technique.

	N1	N2	N3	N4	N5	mean	rank
N1		0.911	0.820	0.922	0.738	0.848	4
N2	0.911		0.734	0.835	0.661	0.785	2
N3	0.820	0.734		0.884	0.834	0.818	3
N4	0.922	0.835	0.884		0.770	0.853	5
N5	0.738	0.661	0.834	0.770		0.751	1

For RCI method was chosen 1 000 simulations, where in each simulation were chosen seven random lights and their strategies. The RCI with STD of the normalisation technique is in Table 4.12.

Table 4.12: RCI and standard deviation of each normalisation technique and rank.

	RCI	rank	STD	rank
N1	0.7437	5	0.1223	4
N2	0.6179	2	0.1876	5
N3	0.6226	3	0.0024	2
N4	0.7462	4	0.0220	3
N5	0.3785	1	0.0013	1

N5 and N2 are not considered as the chosen normalisation techniques, because of their low ranking in 3 out of 4 criteria. N3 has always worse ranking than N1 and N4. The best ranking criteria are N1 and N4. Even though the N4 is better in 3 out of 4 criteria, the values are close. N1 has significantly better results of STD. CoolGrow C11 with constant intermittent is the optimal lamp for lettuce based on 4 normalisation techniques N1, N2, N3 and N4. The weights for entropy method are $[0.15, 0.15, 0.14, 0.15, 0.15, 0.15, 0.11]$.

The ranks of normalisation technifies and data weighted by entropy method for strawberries can be seen in Table 4.13. The weights for strawberries are $[0.14, 0.154, 0.14, 0.15, 0.15, 0.15, 0.13]$. The chosen lamp is CoolGrow C13 with constant intermittent strategy. The weighs of the entropy method are very close to the weight of the equal weight method. The cause is, that lights parameters have similar values with low span of possible values. The ranks and results of entropy and equal weight method sre identical.

For the ranking methods, one rank of criteria is chosen to show the results. The lower is the rank, the more important is the criterion. There is an assumption that the colour is the most important, because it has influence on crop quality and low crop quality decreases demand. Therefore, the rank is set to 1. In Table 4.8 and 4.7, IP class values are 5 or 6 for all light. Because there are only two variants for

Table 4.13: Ranks of each normalisation method for strawberries.

	Spearman	rank	Kendal	rank	RCI	rank	STD	rank
N1	0.9536	4	0.8477	4	0.7922	5	0.1223	4
N2	0.9086	1	0.7854	2	0.6573	3	0.1876	5
N3	0.9366	3	0.8180	3	0.6293	2	0.0024	2
N4	0.9593	5	0.8528	5	0.7841	4	0.0220	3
N5	0.9098	2	0.7506	1	0.3993	1	0.0013	1

IP class, the rank for this criterion is set to the least import one, which is 7. This assigning of ranks takes the possible permutation of ranks from 7! (5040) to 5! (120). The warranty and lamp wear have similar meaning. Most of the lights in the Pareto-optimal set have a similar warranty around 5 years, so the warranty is assigned with the second least important rank, which is 6. Lamp wear can also influence crop quality and energy consumption after some time. Therefore, the lamp wear has the second most important rank, i.e., 2. Efficacy and daily costs can be understood in a similar way. However, daily costs are influenced by light size and beam span, therefore it is set to be more important than efficacy. The last ranks need to be assigned to initial and daily costs. The average ratio of initial costs to daily costs for strawberries is 4,362 days, which is nearly 16 and a half years. If it is assumed that the light are used for a longer period of time, the daily cost will be more important than the initial costs of lights.

Table 4.14: Ranks of each criterion for ranking weight methods.

initial costs	daily c.	lamp wear	IP	warr.	efficacy	colour
4	3	2	7	6	5	1

The results of normalisation techniques for rank weights for romaine lettuce can be seen in Tables 4.15 and 4.16. For reciprocal methods are the weights $[0.15, 0.11, 0.07, 0.25, 0.21,$
$0.18, 0.04]$. The optimal light for this method is again CoolGrow C11 for N1-N4 with constant intermittent strategy. Based on normalisation technique N5 is optimal the same lamp but with dynamic intermittent strategy. For the sum weight method, the weights are $[0.15, 0.19, 0.21, 0.04, 0.07, 0.11, 0.25]$. The optimal light for N1, N2, N4 is CoolGrow C11 with constant intermittent strategy. For N3 was chosen constant intermittent strategy with light Arize A6 and for N5 it is Arize A4 with dynamic intermittent strategy. For strawberries is optimal light in all method CoolGrow C13 with constant intermittent strategy and for N5 with dynamic strategy.

Table 4.15: Ranks of each normalisation method of order reciprocal ranking for romaine lettuce.

	Spearman	rank	Kendal	rank	RCI	rank	STD	rank
N1	0.9534	5	0.8595	5	0.8702	5	0.1164	4
N2	0.9252	2	0.8181	2	0.7667	3	0.2103	5
N3	0.9353	3	0.8183	3	0.7000	2	0.0019	2
N4	0.9500	4	0.8495	4	0.8367	4	0.0186	3
N5	0.8786	1	0.7090	1	0.2748	1	0.0011	1

Table 4.16: Ranks of each normalisation method of ranking sum weight method for romaine lettuce.

	Spearman	rank	Kendal	rank	RCI	rank	STD	rank
N1	0.9592	4	0.8602	4	0.9199	5	0.1405	4
N2	0.9084	1	0.7856	2	0.6863	2	0.1756	5
N3	0.9496	3	0.8419	3	0.8302	3	0.0038	2
N4	0.9631	5	0.8623	5	0.9099	4	0.0289	3
N5	0.9113	2	0.7656	1	0.5053	1	0.0023	1

4.6.5 Summary of results

The results of the weighted sum method can be summarized as follows. For given types of data was not appropriate to use logarithmic normalisation technique, since it always has different optimal light and lower standard deviation. In most cases, CoolGrow LED lights were the best option to use in IVFs. There is a possibility that the given price for CoolGrow light was underestimate and the price for Philips light was overpriced. The strawberries have high demand on intensity, which makes the Pareto-optimal set narrower. The weights for entropy method are very similar to equal weight method. It could be caused by similar span of normalised criteria.

4.7 Other methods for finding Pareto-optimal set and MCDM

For optimisation problems with higher amount of alternatives exist various methods to find Pareto-optimal set of solution. The classic method is the brute force algorithm. Between commonly use heuristic algorithms belongs genetic algorithms like NSGA-II [53] or swarm particle optimisation like MOPSO [54]. Because the data set is small, it is not necessary to use heuristic algorithms to find Pareto-optimal set of solution

The MCDM include many optimisation techniques, where the most common are ϵ-constraint scalarization, Elimination and Choice Translating Reality (ELEC-

TRE), Analytic Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). The SAW method also belongs to MCDM methods, which finds the only one solution. The SAW method was chosen because it is the most basic method and the efficiency of oslutions is described in details.

Summary 5

Indoor vertical farming is a technique to grow crop in fully isolated environment. The highest annual expenses of IVFs are for energy used for lighting. It motivates the presented Master's thesis to focus on lowering lighting costs by experimenting with different lighting strategies and found the optimal light with respect to crop. The focus is on IVFs that are ecological and use renewable energy, which is traded on day-ahead energy markets.

The thesis begins by discussing crop conditions, such as intensity and color, in Chapter 2. This chapter also provides information about different types of LED lights. For top lighting, the number of lights is calculated based on the distance between the lights and the floor. This calculation does not consider different LED light distribution curves or light reflections. The optimal distance between the lights and the floor is determined to minimize both initial and daily illumination costs. Additionally, energy consumption and light degradation calculations are described. Light degradation at the end of the day is the same for both models.

The Chapter 3 begins with introducing day-ahead energy prices of electrical energy and lighting strategies. The focus is on constant and dynamic strategies. In constant strategy is the same intensity during whole time. In dynamic strategy is the intensity different each hour based on the day-ahead energy prices. The aim is to find optimal time and intensity of light with given bounds. These bounds are given by crop and power net. The strategies can have the light on continuously (continuous strategy) or with pauses between each lighting period (intermittent strategy). The last part of this thesis shows implementation in application from company Siemens.

The last Chapter 4 cover multicriterial optimisation, which aims to find optimal strategies and LED lights based on crop. First, the optimisation criteria are introduced. The criteria consist of initial costs, daily costs, lamp wear, IP class, warranty, efficacy and colours of light. The chapter shows results for lettuces and strawberries. Lettuces need longer time of lighting with lower intensity and strawberries need short period of lighting with high intensity and red colour for fruiting. It was found out that brute force algorithm is sufficient algorithm for finding the optimal set of Pareto-optimal solutions and that the solution are even strictly Pareto-optimal. Optimal lighting strategies involve mostly intermittent lighting instead of continuous one and the constant strategy is preferred over dynamic one. The possible cause is that for dynamic strategy needs to be in IVFs more lights, and it increases the initial costs. After finding the Pareto-optimal set of lights, different normalisation techniques and weight methods are investigated to find one optimal lighting strategy and light. It was found that the normalisation techniques, except the logarithmic, has the same results and for strawberries were the result identical even for different weights.

The thesis could be extended by more accurate information about lights, such as distribution curves and light degradation. To minimise the cost for lighting even more, instead of the possibility that the light can be switched on only every quarter of a hour, they can be switched on any time. There has to be taken into account that the result of the work depend on chosen day-ahead prices. For general results, would be needed to done more research based on country or type of renewable energy. Even if there is research on the influence of constant or degradation light and various spectrums on crop, there is no research on changes of intensity during one day. More investigation into effects of light on crop are needed to be done to involve less costly lighting strategies.

Thesis attachment A

The thesis includes PDF files with exchange rates, prices of lights, and their manuals. The files for Siemens applications can also be found here. The `day ahead prices example.xlsx` file consist of few examples of day-ahead energy prices from Nord Pool market. There are specifically 80 prices from the Netherlands from 2023-07-02 to 2023-09-19 (2023-09-19 are cells "C3:C26", 2023-09-18 are cells "D3:D26", ...). The Matlab files are described in order that they are used in chapters. The files are

- `lighttoground.m`

  Figures of number of light and daily energy costs for different distance between light and floor.

- `conversion_to_MWh.m` (function)

  File consist of function which calculate volume of electrical energy that is consumed each hour in MWh and number of lights that are needed for vertical farm. This function is called by `lighttoground.m` and `model_code.m`. The input parameters are vector of HLI, cultivation area length and width, light length, the distance between light and canopy, nominal power of light and PPF of light.

- `model_code.m`

  File that run constant and dynamic optimisation based on farmer conditions. Also shows estimation of degradation for constant and dynamic model.

- `resultMatrixLettuce.m` and `matrixLettucemin.mat`

  Matlab's function that involve all the light parameters and returns matrix of objective functions for romaine lettuce `matrixLettucemin.mat`.

- `resultMatrixStrawberries.m` and `matrixStrawmin.mat`

  Matlab functions that involve all the light parameters and returns matrix of objective functions for strawberries `matrixStrawmin.mat`

- `bruteforce.m`

  Brute force algorithms that find a set of Pareto-optimal solutions.

- `bruteforcefunction.m` (function)

  Brute force algorithms in function to find Pareto-optimal fronts. Called by `philipsprice.m`

- `philipsprice.m`

  File generate figure of the Pareto optimal fronts for Philips lights.

- `multicriterial.m`

  File includes weighted sum method.

- `calculate_RCI.m` (function)

  Calculation of RCI. Called by `multicriterial.m`.

- `lightdistance.m` (function)

  Called by `resultMatrixLettuce.m` and `resultMatrixStrawberries.m`. The function calculates the distance between the light and the canopy or floor.

- `resultInput.m` (function)

  Called by `resultMatrixLettuce.m` and `resultMatrixStrawberries.m`. The function calculates the results of the objective functions.

- `multicriterialInput.m` (function)

  Called by `resultInput.m`. The function calculates the optimal price for energy for 1 day based on the strategy.

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List of Figures

List of Tables


za jednotku času pro x∈⟨x0, x0+h⟩
náboje, tj. proud : q'(t0) = i(t0). c) Hmotnost radioak
Normála grafu funk
lokálního minima (resp. maxim