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TUNING OF COMPLEMENTARY FILTER ATTITUDE ESTIMATOR USING PRECISE MODEL OF MULTICOPTER

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Abstract:

This contribution deals with attitude estimation algorithm based on measuring angular rate, earth magnetic field and specific force and with the tuning of its parameters. At first, sensors and two main methods usually used for attitude estimation are described. Subsequently the particular form of complementary filter attitude estimator is shown. Then the tuning of algorithm's parameters using brute force simulation is described. For this trajectory generated using precise model of multicopter was used. Testing trajectory reflects usual mission of unmanned aerial vehicles called waypoint navigation. Use of trajectory generated by model is convenient because of the knowledge of the true state variables, so the comparison through RMSE (Root Mean Square Error) value is possible.

1. INTRODUCTION

Attitude estimation is a crucial part of any autonomous aerial system. Usually a typical attitude measuring device consists of cheap tri-axis MEMS gyroscope, accelerometer and magnetometer. All these sensors are usually noisy and highly biased. The complex algorithms are needed for optimal attitude estimate. Any existing method for attitude estimation has some parameters to set. These parameters are usually determined by analytical computation or by try and error approach. Considering a real aerial system try and error approach is useless, but with use of mathematical model this approach can quickly lead to accurate results.

In this paper the tuning of parameters is done on simulated trajectory which reflects the traditional use of aerial system – waypoint navigation. The sensor values are distorted by white noise and constant bias to simulate real sensor characteristics. This enables smooth transition of algorithm from simulation to real hardware with real sensors.

2. SENSORS USED FOR ATTITUDE ESTIMATION

Tri-axis MEMS gyroscopes, accelerometers and magnetometers are the most widely used sensors for attitude estimation. In next subsections the type of attitude information available in each sensor is mentioned.

2.1 Gyroscopes

The tri-axis gyroscope measures three components of angular rate with respect to inertial frame expressed in so called body frame (linked with examined object). If the initial attitude is known, the time evolution of attitude can be computed by integrating the angular rate data. This is enough for navigation grade gyroscopes, which can provide sufficient attitude information (error less than 1° in each Euler angle) for more than 10 hours after initialization (depends on particular sensor and implementation of integration). Because of the noise and the bias which is more or less present in all angular rate sensors based on MEMS technology the error grows quickly (usually more than 10°/min) in time. Using cheap MEMS gyroscopes alone for attitude determination is obviously insufficient.

2.2 Accelerometers

The accelerometers measure specific force acting on the examined object. If the object is not moving (or uniformly moving), the accelerometers measure vector of Earth gravitational field which shows us local vertical direction. This information can be used for computing Roll and Pitch Euler angles directly (Euler angles – one of the possible attitude representations [1]) or as a vector measurement known in both reference and body frame. If any force different from reaction to gravitational one acts on the examined body, the information is useless during this period.

2.3 Magnetometer

The magnetometer measures magnetic field, if there is no local source of magnetic field, this sensor measures Earth magnetic field which in short time and position horizon provides constant vector. If we know the size and direction of the magnetic vector in reference frame, then by measuring magnetic vector in body frame we get same type of information like in accelerometer case. This information is relevant unless the magnetic field is disturbed by local magnetic sources.

Each of above mentioned type of sensor provides some kind of information regarding attitude.

However this information alone is not usable for long time attitude estimation with bounded error. Usually these three types of sensors are used together and some sophisticated algorithm uses the advantages of each sensor and combines the information to provide the best attitude estimate. The most used attitude estimation methods utilize some special type of Kalman filter [2] or Complementary filter [3]. Each of these methods will be briefly introduced in the following chapter.

3. METHODS USED FOR ESTIMATION – BASIC PRINCIPLES

In this chapter only the basic principles of each method will be described. These principles and characteristics are more or less general and are not used only for attitude estimation.

3.1 Complementary Filter

Complementary filter is a filtering technique in frequency domain. Two or more sensors, which provide some state variables of measured system, are considered as an input. From each sensor, only a part of frequency spectrum is used and all sensors together cover all spectrum. This means that one sensor complements other in frequency domain, thus the name complementary. The block scheme of complementary filter is depicted in Fig. 1.

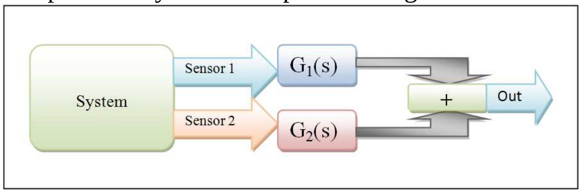

Fig. 1 Principle of Complementary filter

If we have two sensors of the same state variable this condition for filter G1 and G2 should be satisfied:

$$G_1(s)+G_2(s)=1\,. \qquad (1)$$

The Complementary filter is widely used mainly because of the ease of implementation and for its simplicity (only one parameter – crossover frequency is required for two sensor case).

3.2 Kalman Filter

Kalman filter is a well known estimation technique developed in 1960 [4]. It is primarily developed for estimating the state of linear systems with additive Gaussian white noise and with noisy measurements. If we know the characteristic of all noises, the Kalman filter algorithm guarantee the optimality of its state estimate. The iterative discrete algorithm consists of two steps, the prediction step and the correction step. The individual steps of Kalman filter are:

Prediction step:

$$\mathbf{x}(k+1\,|\,k)=\mathbf{A}\cdot\mathbf{x}(k)+\mathbf{B}\cdot\mathbf{u}(k) \qquad (2)$$

$$\mathbf{P}(k+1\,|\,k)=\mathbf{A}\cdot\mathbf{P}\cdot\mathbf{A}'+\mathbf{R}\,. \qquad (3)$$

Update step:

$$\mathbf{K}=(\mathbf{P}(k+1\,|\,k)\cdot\mathbf{C}')\cdot inv(\mathbf{C}\cdot\mathbf{P}(k+1\,|\,k)\cdot\mathbf{C}'+\mathbf{Q}) \qquad (4)$$

$$\mathbf{x}(k+1)=\mathbf{x}(k+1\,|\,k)+\mathbf{K}(\mathbf{z}(k)-\mathbf{C}\cdot\mathbf{x}(k+1\,|\,k)) \qquad (5)$$

$$\mathbf{P}(k+1)=\left(\mathbf{I}-\mathbf{K}\cdot\mathbf{C}\right)\cdot\mathbf{P}(k+1\,|\,k)\,. \qquad (6)$$

Where $\mathbf{x}(k)$ is a state vector at k-th iteration, **A** is the system matrix, **B** is the input matrix, **C** is the measurement matrix, **u** is the vector of system inputs, **z** is the vector of measurements, **P** is the system covariance matrix, **R** and **Q** are covariance matrices of system resp. measurement noises.

For attitude estimation the angular rate is usually considered as an input to the system and accelerometer and magnetometer values as the measurements. The system equations describing the attitude estimation problem are non-linear. Since the original Kalman filter is designed only for linear systems some suboptimal adaptation for non-linear systems were developed. The most common is EKF (Extended Kalman Filter) which uses first order Taylor expansion in every iteration.

Regarding the attitude estimation, Kalman filter is more difficult to use than the complementary filter. There are more parameters to tune (system and measurement noise covariance matrices) and the whole algorithm is computationally very expensive and the implementation to the target device with microcontroller needs lot of effort in comparison with complementary filter.

4. COMPLEMENTARY FILTER ATTITUDE ESTIMATOR

If we take into account the principle of complementary filter along with characteristic of individual sensors mentioned in introduction section, it is beneficial to use only high frequency component of gyroscope sensor and low frequency component of the remaining sensors. The implementation of complementary filter can have different forms, but the basic principle of frequency filtering is still present. Hereafter mentioned algorithm is modified complementary filter attitude estimator from [3].

The rotation matrix is used for internal attitude representation. Rotation matrix is the only unique and non-singular attitude representation [1]. The only disadvantage is the number of elements - 9 (quaternion – 4, Euler angles - 3). There exist simple relations between all these attitude representations. The core of the algorithm is the discrete equation integrating the gyroscope sensor values compensated for so-called bias using the rotation matrix R:

$$\mathbf{R}_{n+1}=\mathbf{R}_n\cdot(\mathbf{I}+\mathbf{\Omega}_n\cdot\Delta T) \qquad (7)$$

$$\mathbf{\Omega} = \begin{bmatrix} 0 & -(\omega_z - b_z) & (\omega_y - b_y) \\ (\omega_z - b_z) & 0 & -(\omega_x - b_x) \\ -(\omega_y - b_y) & (\omega_x - b_x) & 0 \end{bmatrix}, \quad (8)$$

where ΔT is sampling period, n is iteration index, $\mathbf{I}$ is identity matrix and $\boldsymbol{\omega}$ is vector of angular rates (expressed in body frame). The information from accelerometer and magnetometer are passed to the core of the algorithm through the bias estimate:

$$\mathbf{b}(n) = k_p \cdot \mathbf{e}_n + k_s \sum_{i=1}^{n} \mathbf{e}_i \quad (9)$$

$$\mathbf{e} = v \cdot \mathbf{e}_a + k_m \cdot \mathbf{e}_m \quad (10)$$

$$\mathbf{e}_a = (\mathbf{R}' \cdot \frac{\mathbf{g}_I}{\|\mathbf{g}\|}) \times (\frac{\mathbf{a}_B}{\|\mathbf{a}\|}) \quad (11)$$

$$\mathbf{e}_m = (\mathbf{R}' \cdot \frac{\mathbf{m}_I}{\|\mathbf{m}_I\|}) \times (\frac{\mathbf{m}_B}{\|\mathbf{m}_B\|}) \quad (12)$$

$$v = \exp\left(\frac{-(\|\mathbf{a}\| - g)^2}{\sigma}\right), \quad (13)$$

Where k_p , k_s , k_m and σ are parameters of complementary filter, $\mathbf{g}$ is gravitational acceleration vector, $\mathbf{a}_B$ is measured specific force, $\mathbf{m}$ is Earth magnetic vector, subscript I means reference value expressed in inertial frame and subscript B means measured value expressed in body frame. The terms in (11) and (12) are angular rate vectors expressed in body frame, which would lead to alignment of reference and measured vectors. Term (13) causes that specific force vector is used only when its magnitude is close to value of Earth gravitational acceleration, so it avoids using information from accelerometer when it is irrelevant. Information from magnetometer is considered to be relevant all the time (this assumption can be violated easily in indoor environment). As term (9) is suggested direction of rotation leading to alignment (reference vectors with measured vectors) multiplied by constant it is clear that this vector (bias estimate) is passed back to the core algorithm:

$$\boldsymbol{\omega} = \boldsymbol{\omega}_B \text{-} \mathbf{b}, \quad (14)$$

Where $\boldsymbol{\omega}$ is vector of angular rates from (8), $\boldsymbol{\omega}_B$ is measured angular rate by gyroscope and $\mathbf{b}$ is bias estimate from (9). Sum of all previous errors in (9) allows having zero stable error because of the same principle as S term in PSD controller.
Since all computations are usually performed on computer, care has to be taken to rotation matrix. Rotation matrix is a special orthonormal matrix (all rows or columns are orthogonal and perpendicular vectors). This property is slightly violated in each iteration. Without correction, this would lead to divergence of rotation matrix and of whole algorithm.

One of the possible orthogonalization equations is [5]:

$$\mathbf{R} = {}^{3}\!/_{2}\mathbf{R} - {}^{1}\!/_{2}\mathbf{R} \cdot \mathbf{R}' \cdot \mathbf{R}, \quad (15)$$

Equations (7)-(15) form the complete complementary filter algorithm for attitude estimation using gyroscope accelerometer and magnetometer.

5. COMPLEMENTARY FILTER PARAMETERS TUNING

The complete complementary filter algorithm has 4 parameters in total. Using these parameters we can control the behavior of the filter, namely bias settling time (k_s), vector following speed (k_p), magnetometer weight (k_m) and rejection of acceleration (σ). If we set these values randomly, the filter can operate unexpectedly and can diverge. Therefore it is convenient to do parameters tuning in simulation environment. For this purpose the filter was tested in MATLAB environment.

5.1 Simulated sensors

To perform a simulation of complementary filter we need all filter inputs namely angular rate vector, specific force vector and magnetic field vector. All these values were generated using precise model of multicopter. The reason for using this model is to have a trajectory which is characteristic for this type of unmanned aerial vehicle. The model outputs true values of the variables needed for testing the complementary filter algorithm. Noise typical for MEMS sensors is added to these variables to precisely simulate the real world conditions. The simulated sensor values are sampled with the rate of 100 Hz. The static outputs of one axis of simulated gyroscope and real STMicroelectronics L3G4200 gyroscope are compared on Fig. 2.

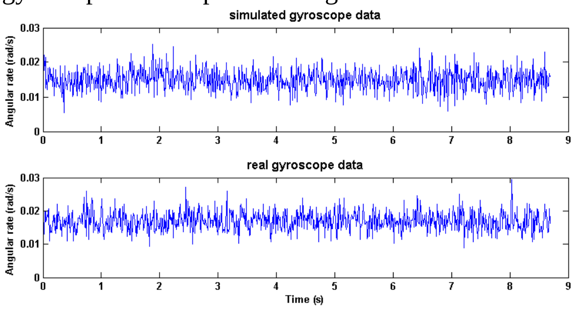

Fig. 2 Simulated and real static gyroscope outputs

5.2 Testing trajectory

Testing trajectory is based on a standard waypoint navigation mission. This is the typical trajectory which will be flown by the system so it is reasonable to tune the parameters on this trajectory. The trajectory waypoints are listed in the following table:

Table 1: Waypoints of testing trajectory

Waypoint	*X[m]*	*Y[m]*	*h[m]*	*Comment*
Start	0	0	0	hover 10 s
1.	25	-25	5	
2.	30	30	10	
3.	30	-30	30	
4.	-30	30	30	
5.	30	40	30	
6.	-30	40	15	
Stop	0	0	5	hover 20s

The top view and height profile of the trajectory is on Fig. 3 and Fig. 4 respectively.

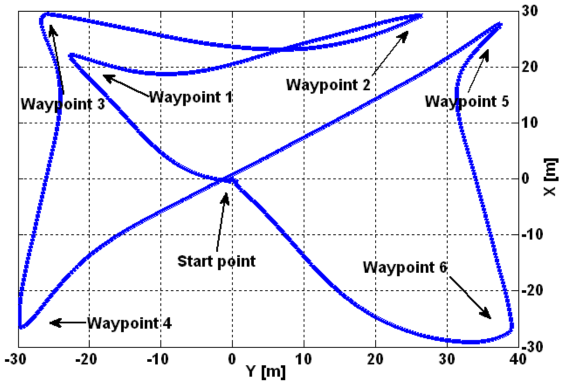

Fig. 3 Top view of testing trajectory

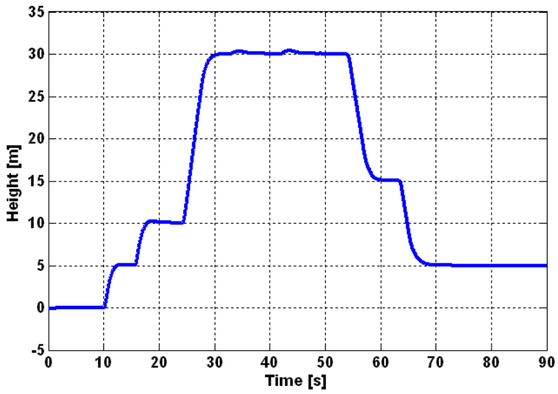

Fig. 4 Height profile of testing trajectory

The simulated flight lasted 90 s. The coordinated system is defined as NED (x-North, y-East, z-Down). The magnetometer is in this simulation considered to measure only Earth's magnetic field located near equator. No magnetic field disturbances are simulated in this trajectory. This should be taken into account since the Earth magnetic field can be easily disturbed mainly in indoor environment. The simulated sensor values for gyroscope, accelerometer and magnetometer are on Fig. 5, Fig. 6 and Fig. 7 respectively.

5.3 Tuning approach

The tuning of parameters was performed using brute-force. In three steps an arrays of parameters was generated. In a given step for each combination of parameters the estimation of attitude for whole trajectory is computed and compared with true simulated Euler angles (which are known thanks to simulation).

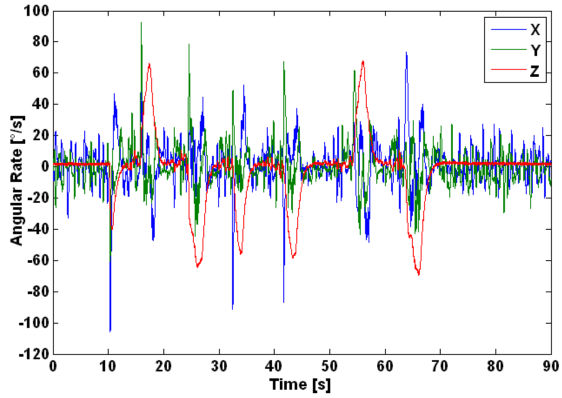

Fig. 5 Simulated gyroscope values for testing trajectory

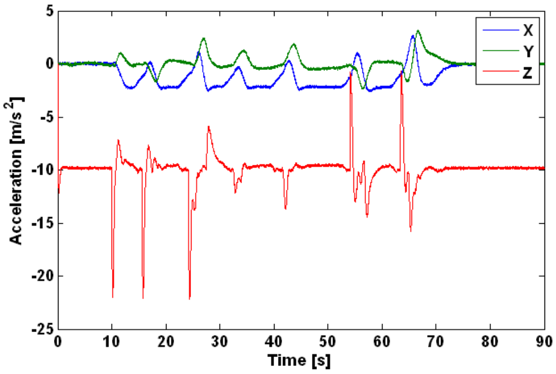

Fig. 6 Simulated accelerometer values for testing trajectory

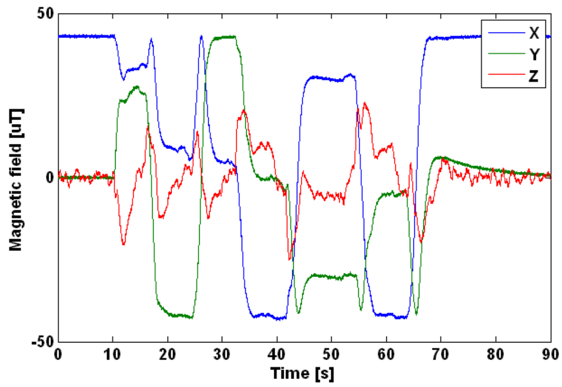

Fig. 7 Simulated magnetometer values for testing trajectory

The sets of parameters are compared using RMSE value. This value is computed using the true and estimated attitude for testing trajectory:

$$\mathrm{RMSE} = \sum_{i=1}^{N} \left[\delta\phi(i)^2 + \delta\theta(i)^2 + \delta\psi(i)^2 \right], \tag{16}$$

where i is the sample index, Φ, θ and ψ are roll, pitch and yaw Euler angles respectively and the error operator δ is defined as:

$$\delta\phi = \phi_T - \phi_M \,, \tag{17}$$

and index T denotes true value and M the estimated value. The set of parameters with lowest RMSE value are considered as the best from the particular array.

The tuning is focused for all parameters except the magnetometer weight (k_m). This value is set to constant ($k_m = 1$) for all tuning since large values gives better results because of the almost ideal vector measurements but this can lead to very poor real world results when using in place with local magnetic disturbances. The whole tuning is based on the assumption of the existence of one global minimum. This assumption cannot be guaranteed but the further mentioned results show very good performance with parameters tuned using this approach. The complementary filter runs at the rate of 100 Hz the same rate the sensors outputs their values.

In the first step the rough tuning was performed to find the order of each parameter in which the filter operates well. The possible values for each parameter are shown in the following table.

Table 2: Values of parameters for first rough tuning

Parameter	***Values***
k_p	100, 10, 1, 0.1, 0.01
k_i	1, 0.1, 0.01, 0.001, 0.0001
σ	1, 0.1, 0.01, 0.001, 0.0001

In the first step there are 125 possible combinations in total and for each combination estimated attitude for whole trajectory is computed in order to obtain RMSE value. The best parameter set with the lowest RMSE value is shown in Table 3.

Table 3: First step best parameters set

Parameter	k_p	k_i	σ	RMSE
Value	1	0.001	0.001	10.63

In the second step, the finer tuning with intervals around the best parameters from previous tuning is done. Table 4 and Table 5 again show the parameter values and best parameter set respectively.

Table 4: Values of parameters for second finer tuning

Parameter	***Values***
k_p	5, 2.5, 1, 0.5, 0.25
k_i	0.005, 0.0025, 0.001, 0.0005, 0.00025
σ	0.005, 0.0025, 0.001, 0.0005, 0.00025

Table 5: Second step best parameters set

Parameter	k_p	k_i	σ	RMSE
Value	1	0.0025	0.0005	6.35

In the last step the finest tuning is performed. The values chosen for individual parameters are listed in Table 6. As you can see in Table 7, where the best parameter set is listed, the RMSE value decreased slightly thus finer tuning is not necessary. Additionally the values of parameters lie in the middle of their possible values which means that the best set lies very close to the local minimum of RMSE.

Table 6: Values of parameters for third finest tuning

Parameter	***Values***
k_p	1.3, 1.2, 1.1, 1, 0.9, 0.8, 0.7
k_i	0.0035, 0.003, 0.0025, 0.00020, 0.00015
σ	$(2, 3, \ldots 7, 8)\cdot 10^{-4}$

Table 7: The best parameters set

Parameter	k_p	k_i	σ	RMSE
Value	0.9	0.002	0.0006	6.05

On Fig. 8, Fig. 9 and Fig. 10 there are comparisons of true and estimated roll, pitch and yaw Euler angles which were computed using the complementary filter with the parameters from Table 7.

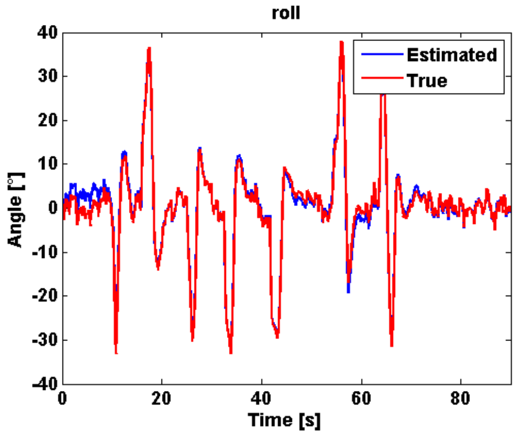

Fig. 8 True and estimated roll Euler angle for testing trajectory using the best parameters set

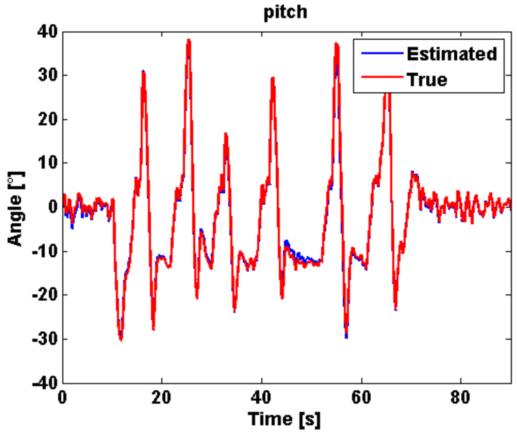

Fig. 9 True and estimated pitch Euler angle for testing trajectory using the best parameters set

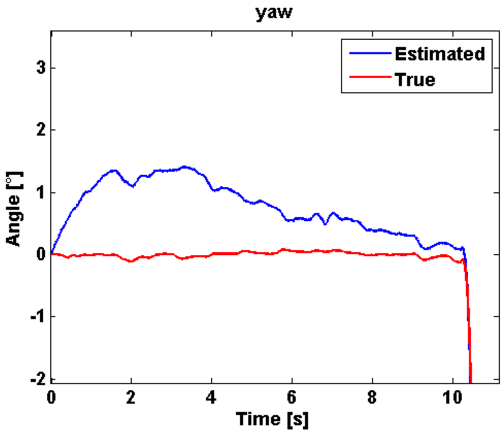

Fig. 10 True and estimated yaw Euler angle for testing trajectory using the best parameters set – detail of bias settling

6. CONCLUSION

In this article the complementary filter algorithm for attitude estimation was presented and the method for tuning of its parameters was shown. In the section 4 all equation of complementary filter are mentioned. The main contribution is parameter tuning using trajectory generated by precise model of particular UAV (hexacopter). The tuning was performed iteratively in three steps. Results of each step are in Table 3, Table 5 and Table 7. The performance of complementary filter for attitude estimation with parameters tuned using proposed approach can be seen on Fig. 8, Fig. 9 and Fig. 10 where true and estimated Euler angles for testing trajectory are plotted. The tuning approach provides very simple and fast way how to implement and get familiarly with complementary filters for attitude estimation.

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