Practical Construction of Reflectances for Spectral Rendering

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ABSTRACT

We propose a method to construct practical reflectances for spectral rendering. A set of 1400 real reflectances are measured and more spectra with higher saturated colors are generated using Bouguer's law. These spectra serve as the basis for constructing the spectra for any colors. Given a color triplet P, in the color space, we find a small local tetrahedron that contains P and the reflectances at the vertices are known. Thus the spectrum at P is constructed using trilinear interpolation. Our derived spectra correspond closely to real spectra and the transformation works for a large volume of color space. This method of spectral construction has been applied to texture mapping and simulation of wavelength-dependent phenomena in spectral rendering.

Keywords

Spectra, color, reflectance, transformation, spectral rendering.

1. INTRODUCTION

RGB triplets are commonly used to model light and objects in computer graphics. Although efficient, this approach is insufficient in accuracy [Hal83a, Hal89a] and is incapable of rendering wavelength-dependent phenomena such as light interference and diffraction [Nas83a]. To eliminate the drawbacks, considerable researches have been developed on spectral rendering [Gon94a, Smi90a, Sta99a, Sun00a]. The spectral approach models light and objects with spectra and synthesizes a spectral image that can be converted to an RGB image for display.

The spectra of many light sources [Gla95a, Wys82a] and materials [Kri53a, Vrh94a] have been measured. Since a large number of illuminators and materials exist in reality, some spectra needed in spectral rendering may not be available. In this case, one needs to construct spectra with the information of color components. The transformation from colors to spectra also enables one to use RGB resources (such as RGB images and textures) in spectral rendering.

Because the color-spectrum transformation is one-to-many, we must constrain the derived spectra so that they can be determined uniquely. The past solution assumes the derived spectra as a linear combination of three basis functions. Simple functions such as delta [Gla89a], box [Hal89a], sinusoid [Wan87a], and Gaussian [Sun99a] have been proposed to work as the basis functions. However, the spectra derived in this way suffer from the limitation that they have an artificial functional structure and may differ substantially from the real spectra.

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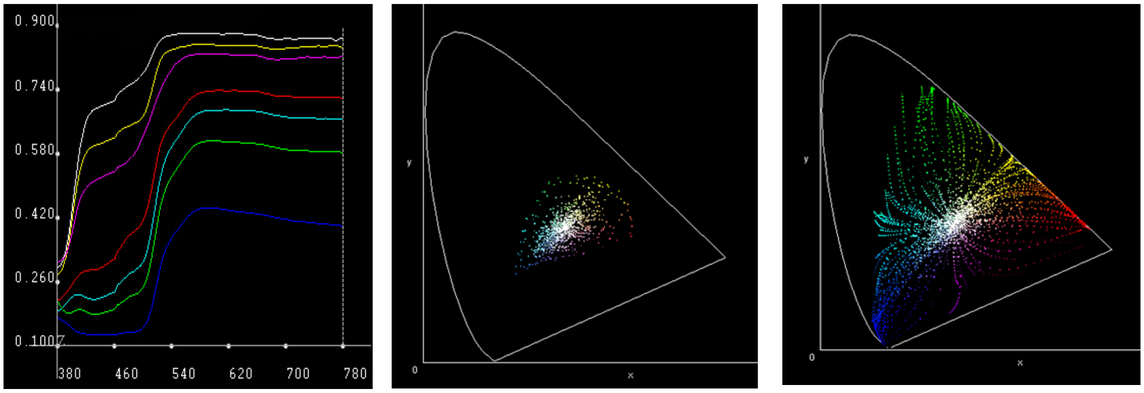
This paper proposes a new method to construct practical reflectances for spectral rendering. First, we measure a set of real reflectances. From the measured spectra, we generate more spectra that have higher color saturation using Bouguer's law. The measured and Bouguer-generated spectra span a large volume in the color space, and their union is used to serve as the *base spectra* for constructing reflectance for any color. Given a color triplet P, in the CIE XYZ color space, we find a small tetrahedron such that it contains P and the reflectances at the vertices are given by the base spectra. Thus the spectrum at P can be constructed using trilinear interpolation of the tetrahedron vertices. The derived spectra correspond closely to real spectra and the transformation works for a large volume of color space. We will apply this spectral construction to spectral rendering including texture mapping and diffraction simulation.

2. SPECTRA AND COLORS

A spectrum describes a physical quantity in the visible range, roughly from 400 to 700 nm. Important spectra include spectral power distributions (SPDs) of light and reflectances of surfaces. As the perception of light by the human visual system, colors have three dimensions. The tristimulus values of a color in the CIE XYZ model can be calculated from the following integrals in the visible range:



(a) (b) (c)

Figure 1: (a) Measured spectral curves. (b) Color distribution of all measured reflectances. (b) Color distribution of all base spectra (including measured and Bouguer-generated spectra).

$$X_k = \kappa \int I(\lambda)\bar{x}_k(\lambda)d\lambda \, , \; k = 1,2,3 \qquad (1)$$

where κ is a positive constant, $I(\lambda)$ is light spectrum, and $\bar{x}_k(\lambda)$ are the CIE color matching functions. In practice, it is convenient to use the relative values

$$x = X_1/(X_1 + X_2 + X_3) \, , \; y = X_2/(X_1 + X_2 + X_3) \qquad (2)$$

to specify color chromaticity in the *xy*-plane.

Let **R**, **G** and **B** be red, green and blue, respectively.[1] Then any color **P** can be expressed as

$$\mathbf{P} = r\mathbf{R} + g\mathbf{G} + b\mathbf{B} \qquad (3)$$

The specification (*r*, *g*, *b*) is called an RGB model of colors. If *r*, *g*, *b* are non-negative, then the color range represented by (*r*, *g*, *b*) is triangle RGB in the chromaticity xy-plane. (*r*, *g*, *b*) can be computed from (*X*, *Y*, *Z*) through a linear transformation

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = M \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad (4)$$

where matrix *M* is known when the primaries **R**, **G** and **B** are given (see page 585-587 in [Fol96a]).

3. BOUGUER GENERATION

The samples we measured are a set of 1400 color paints. The reflectances are measured using a PR-650 spectroradiometer of PHOTO RESEARCH®. Figure 1(a) shows measured spectral curves and 1(b) displays the color distribution of all measured reflectances in the xy-plane. We see that there is still large region not covered by the measurements.

To enlarge the covered color space, we generate more spectra using Bouguer's law [Eva61a, Mac85a]. This law states that the transmittance for light traveling from one point to another is given by

$$T(\lambda) = 10^{-a(\lambda)l} \qquad (5)$$

where $a(\lambda)$ is the absorptivity of the material and *l* is the length of the light path. Increasing *l* results in an increase in the color saturation [Eva61a, Mac85a]. If we replace $a(\lambda)$ with a measured reflectance $R(\lambda)$ and regard the result as a reflectance

$$R'_l(\lambda) = 10^{-R(\lambda)l} \, , \qquad (6)$$

this generates a series of reflectances with higher saturation when *l* increases. Figure 1(c) include the spectra generated from Eq. (6). The covered color space has been increased substantially.

4. SPECTRAL CONSTRUCTION

The key property upon which this paper is based is the linearity of the transformation from the spectral space to the CIE XYZ space. Suppose that $\mathbf{P}_1$, $\mathbf{P}_2$,…, $\mathbf{P}_n$ are *n* colors and their spectra $I_1(\lambda)$, $I_2(\lambda)$,…, $I_n(\lambda)$ are known. Then for any color given by

$$\mathbf{P} = \sum_{i=1}^{n} k_i \mathbf{P}_i \, , \qquad (7)$$

where k_1, k_2 ,…, k_n are non-negative,

$$I(\lambda) = \sum_{i=1}^{n} k_i I_i(\lambda) \, , \qquad (8)$$

is a spectrum for color **P**. Note that if we require $k_1 + k_2 + \ldots + k_n = 1$, then color **P** is inside the polyhedron formed by $\mathbf{P}_1, \mathbf{P}_2$,…, $\mathbf{P}_n$.

To generate the spectrum for a given color **P**, we just need to find a set of color points $\mathbf{P}_1, \mathbf{P}_2$,…, $\mathbf{P}_n$ such that their spectra are known and polyhedron

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[1] The bold symbols are used to denote vectors.

$P_1P_2...P_n$ encloses **P**. Note that without this enclosure condition, **P** is located outside of polyhedron $P_1P_2...P_n$ and some coefficients of $k_1, k_2, \ldots, k_n$ are negative. This may cause negative values in the generated spectrum, which is not valid.

Given color **P**, there may exist a large number of color polyhedrons that enclose **P**. Our approach is to select a tetrahedron as small as possible. Here is a procedure to find such a tetrahedron:

1. Divide the color space with planes π_1, π_2, and π_3 (Figure 2). These planes pass through **P** and are parallel with the *xy*, *yz*, and *zx*-plane, respectively. Thus the entire space is subdivided into eight subspaces.
2. In each subspace, find from the base spectra (measured or Bouguer-generated) the color point that is closest to **P**. Thus we obtain eight points {**A**, **B**, **C**, **D**, **E**, **F**, **G**, **H**}, each derived from one subspace.
3. Construct tetrahedrons ABCF, ACDH, AEFH, CFGH, and ACFH (Figure 2). If any tetrahedron contains point **P**, it will be used to derive the spectrum of **P** using linear interpolation.

Figure 3a shows the locations of tetrahedron (points with line connection) for a color triplet. Figure 3b shows the constructed spectrum (solid curve) as compared to the spectra (dotted curves) for the tetrahedron vertices.

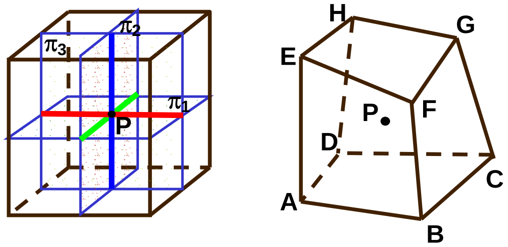
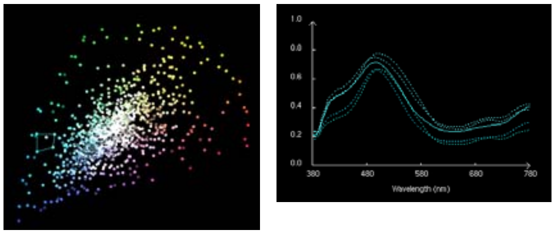


Figure 2: Space subdivision and volume formed by the eight points found in eight subspaces.



(a) (b)

Figure 3: (a) The tetrahedron for given color (in white). (b) The constructed spectrum (solid curve) and the base spectra (dotted curves) for the tetrahedron vertices.

5. RENDERING

Spectral rendering specifies light and material with spectra, retains full wavelength information in modeling light-object interactions, and generates spectral images as synthetic results. A spectral image is similar to an RGB image except that every pixel contains a light intensity instead of an RGB triplet.

When some needed spectra such as reflectances are not available, we should use the spectral construction method to construct the spectra from the information of color components. If the color information is RGB triplet, we need first transform the color from the RGB to the CIE XYZ model. This is the reverse transformation of Eq. (4). Then we just need to follow the steps described in Section 4 to construct the spectra. The same process works for texture mapping that uses RGB textures. Figure 4(a) shows an example of texture mapping in spectral rendering that uses an RGB texture.

Spectral rendering is absolutely necessary for simulating wavelength-dependent phenomena. An important type of such phenomena is light diffraction. For example, the color stripes on a CD ROM are caused by diffraction at the periodic grating on the CD ROM surface. The local illumination behavior can be described by [Sun00a]

$$I(\lambda) = \kappa I_0(\lambda) \sum_{j,l=-\infty}^{\infty} \delta[\frac{a(\mathbf{q}\cdot\hat{\mathbf{s}})}{\lambda} - j]\delta[\frac{b(\mathbf{q}\cdot\hat{\mathbf{t}})}{\lambda} - l], \quad (9)$$

where $\mathbf{q} = \hat{\mathbf{k}}_1 - \hat{\mathbf{k}}_2$ with $\hat{\mathbf{k}}_1$ and $\hat{\mathbf{k}}_2$ are the incident and outgoing light directions, $\hat{\mathbf{s}}$ and $\hat{\mathbf{t}}$ are the grating directions, *a* and *b* are the grating spacings, and κ is a constant. The delta functions in Eq. (9) implies that given the incident and outgoing light directions, only particular wavelengths have non-zero energies in the reflected spectra.

Figure 4(b) shows a rendered CD ROM with its reflection image on a plane. Note that the stripe colors and orientations of the CD ROM and its reflection are different. This is a typical phenomenon of diffraction where the effect depends on the incident and outgoing light directions sensitively. When another object is present in the scene and its reflectance is unknown, we need to retain spectral rendering and construct the unknown reflectance from its color information. Figure 4(c) shows a rendered image for the same CD ROM but with a sphere added to the scene. The reflectance of the sphere is not known, and it is constructed with the method described in Section 4. As result, the image not only retains the color stripes on the CD ROM, but also shows inter-reflections between the sphere and the CD ROM.

(a) (b) (c)

Figure 4: (a) Texture mapping in spectral rendering. (b) CD ROM and its reflection. (b) CD ROM and a sphere whose reflectance is constructed from color information.

6. CONCLUSIONS

We have proposed a method to generate realistic reflectances from color components. This technique is based on measured spectra. We have measured the reflectances of 1400 color paints. To enlarge the span region of the base spectra, we proposed to include spectra of high color saturation that are generated using Bouguer's law.

From the base spectra, we proposed an algorithm to construct the spectrum of any color P. This algorithm first subdivides the color space into eight subspaces with three planes that pass through P and are parallel with the *xy*, *yz*, and *zx*-plane, respectively. For each subspace, we look for the color point which is closest to P and whose spectrum is known. This results in eight points, from which a small tetrahedron containing P can be found.

The spectra constructed with our method correspond closely to the real spectra and the deriving operation works for a large volume of colors. This method has been applied to texture mapping using RGB textures and simulating wavelength-dependent phenomena in spectral rendering.

In this paper we have measured the reflectances of a set of paint colors. In the future such measurements can be done on other sets of materials such as cosmetic colors, textile dyes, plastic colors, and automobile paints. It is also interesting to construct a spectral database in terms of a regular grid in the color space. Using this database, spectral derivation from colors will be very efficient, which is constant in time and independent of the size of the database.

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